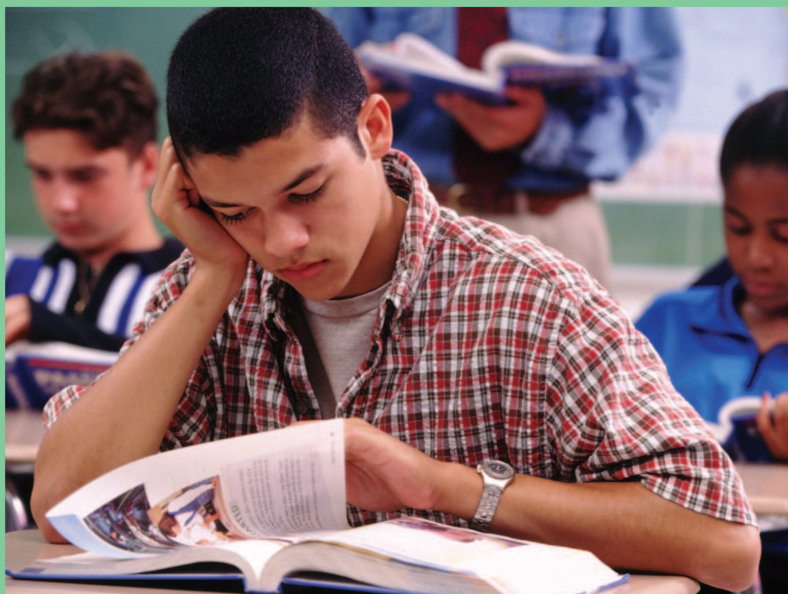
GRADE
9

TAKS STUDY GUIDE

Texas Assessment of Knowledge and Skills



A Student and Family Guide to Grade 9
Reading • Mathematics

TAKS STUDY GUIDE

Texas Assessment of Knowledge and Skills

Grade 9

Reading and Mathematics

A Student and Family Guide

A Letter from the Associate Commissioner of Education

Dear Student and Parent:

The Texas Assessment of Knowledge and Skills (TAKS) is a comprehensive testing program for public school students in grades 3–11. TAKS replaces the Texas Assessment of Academic Skills (TAAS) and is designed to measure to what extent a student has learned, understood, and is able to apply the important concepts and skills expected at each tested grade level. In addition, the test can provide valuable feedback to students, parents, and schools about student progress from grade to grade.

Students are tested in mathematics in grades 3–11; reading in grades 3–9; writing in grades 4 and 7; English language arts in grades 10 and 11; science in grades 5, 10, and 11; and social studies in grades 8, 10, and 11. Every TAKS test is directly linked to the Texas Essential Knowledge and Skills (TEKS) curriculum. The TEKS is the state-mandated curriculum for Texas public school students. Essential knowledge and skills taught at each grade build upon the material learned in previous grades. By developing the academic skills specified in the TEKS, students can build a strong foundation for future success.

The Texas Education Agency has developed this study guide to help students strengthen the TEKS-based skills that are taught in class and tested on TAKS. The guide is designed for students to use on their own or for students and families to work through together. Concepts are presented in a variety of ways that will help students review the information and skills they need to be successful on the TAKS. Every guide includes explanations, practice questions, detailed answer keys, and student activities. At the end of this book is an evaluation form for you to complete and mail back when you have finished the guide. Your comments will help us improve future versions of this guide.

There are a number of resources available for students and families who would like more information about the TAKS testing program. Information booklets are available for every TAKS subject and grade. Brochures are also available that explain the Student Success Initiative promotion requirements and the new graduation requirements for eleventh-grade students. To obtain copies of these resources or to learn more about the testing program, please contact your school or visit the Texas Education Agency website at www.tea.state.tx.us.

Texas is proud of the progress our students have made as they strive to reach their academic goals. We hope the study guides will help foster student learning, growth, and success in all of the TAKS subject areas.

Sincerely,

Ann Smisko

Ann Smisko
Associate Commissioner
Curriculum, Assessment, and Technology
Texas Education Agency

MATHEMATICS

INTRODUCTION

What Is This Book?

This is a study guide to help you strengthen the skills tested on the Grade 9 Texas Assessment of Knowledge and Skills (TAKS). TAKS is a state-developed test administered with no time limit. It is designed to provide an accurate measure of learning in Texas schools.

By acquiring all the skills taught in the ninth grade, you will be better prepared to succeed on the Grade 9 TAKS test and during the next school year. This study guide is organized into two sections. This section is about mathematics.

What Are Objectives?

Objectives are goals for the knowledge and skills that you should achieve. The specific goals for instruction in Texas schools were provided by the Texas Essential Knowledge and Skills (TEKS). The objectives for TAKS were developed based on the TEKS.

How Is the Mathematics Section Organized?

The mathematics section of this study guide is divided into the ten objectives tested on TAKS. A statement at the beginning of each objective lists the mathematics skills you need to acquire. The study guide covers a large amount of material. You should not expect to complete it all at once. It may be best to work through one objective at a time.

Each objective is organized into review sections and a practice section. The review sections present examples and explanations of the mathematics skills for each objective. The practice sections feature mathematics problems that are similar to the ones used on the TAKS test.

How Can I Use This Book?

First look at your Confidential Student Report. This is the report the school gave you that shows your TAKS scores. This report will tell you which TAKS subject-area test(s) you passed and which one(s) you did not pass. Use your report to determine which skills need improvement. Once you know which skills need to be improved, you can read through the instructions and examples that support those skills. You may also choose to work through all the sections. Pace yourself as you work through the study guide. Work in short sessions. If you become frustrated, stop and start again later.

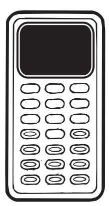
What Are the Helpful Features of the Mathematics Section?

- Look for the following features in the margin:

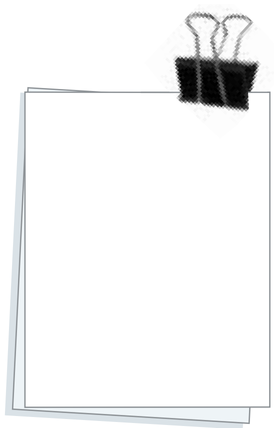
  Ms. Mathematics provides important instructional information for a topic.

Do you see that . . . points to a significant sentence in the instruction.

Calculator suggests that using a graphing calculator might be helpful.



Memo provides page references in this study guide for additional information.



- There are several words in the mathematics section that are important for you to understand. These words are bold-faced in the text and are defined when they are introduced. Locate the bold-faced words and review the definitions.
- Examples are contained inside shaded boxes.
- Each objective has “Try It” problems based on the examples in the review sections.
- A Mathematics Chart for the Grade 9 TAKS test is included on pages 90–91 and also as a tear-out page in the back of the book. This chart includes useful mathematics information. The tear-out Mathematics Chart in the back of the book also provides both a metric and a customary ruler to help solve problems requiring measurement of length.

How Should the “Try It” Problems Be Used?

“Try It” problems are found throughout the review sections of the mathematics study guide. These problems provide an opportunity for you to practice skills that have just been covered in the instruction. Each “Try It” problem features lines for your responses. The answers to the “Try It” problems are found immediately following each problem.

While completing a “Try It” problem, cover up the answer portion with a sheet of paper. Then check the answer.

What Kinds of Practice Questions Are in the Study Guide?

The mathematics study guide contains questions similar to those found on the Grade 9 TAKS test. There are two types of questions in the mathematics section.

- Multiple-Choice Questions: Most of the practice questions are multiple choice with four answer choices. These questions present a mathematics problem using numbers, symbols, words, a table, a diagram, or a combination of these. Read each problem carefully. If there is a table or diagram, study it. You should read each answer choice carefully before choosing the best answer.
- Griddable Questions: Some practice questions use an eight-column answer grid like those used on the Grade 9 TAKS test.

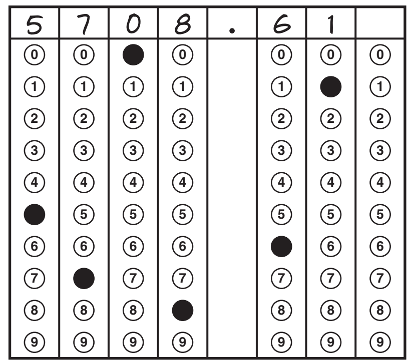
How Do You Use an Answer Grid?

The answer grid contains eight columns, which includes three decimal places: tenths, hundredths, and thousandths.

Suppose 5708.61 is the answer to a problem. First write the number in the blank spaces. Be sure to use the correct place value. For example, 5 is in the thousands place, 7 is in the hundreds place, 0 is in the tens place, 8 is in the ones place, 6 is in the tenths place, and 1 is in the hundredths place.

Then fill in the correct bubble under each digit. Notice that if there is a zero in the answer, you need to fill in the bubble for the zero.

The grid shows 5708.61 correctly entered. The zero in the tens place is bubbled in because it is part of the answer. It is not necessary to bubble in the zero in the thousandths place, because this zero will not affect the value of the correct answer.



Where Can Correct Answers to the Practice Questions Be Found?

The answers to the practice questions are in the answer key at the back of the mathematics section pages 288–305. Each question includes a reference to the page number in the answer key for the answer to the problem. The answer key explains the correct answer, and it also includes some explanations for incorrect answers. After you answer the practice questions, you can check your answers.

If you still do not understand the correct answer after reading the answer explanations, ask a friend, family member, or teacher for help. Even if you have chosen the correct answer, it is a good idea to read the answer explanation because it may help you better understand why the answer is correct.

Grades 9, 10, and 11 Exit Level
Mathematics Chart

LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

Metric and customary rulers can be found on the tear-out Mathematics Chart in the back of this book.

Grades 9, 10, and 11 Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$
Surface Area	cube	$S = 6s^2$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism or cylinder	$V = Bh$*
	pyramid or cone	$V = \frac{1}{3}Bh$*
	sphere	$V = \frac{4}{3}\pi r^3$
**B represents the area of the Base of a solid figure.*		
Pi	π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

MATHEMATICS

Objective 1

The student will describe functional relationships in a variety of ways.

For this objective you should be able to recognize that a function represents a dependence of one quantity on another and can be described in a number of ways.

What Is a Function?

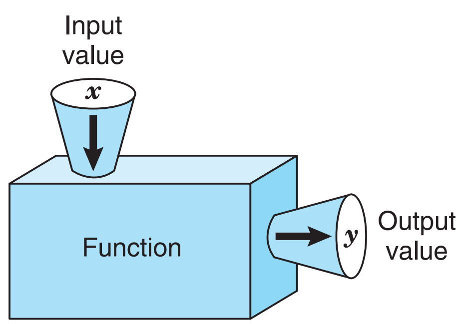
A **function** is a set of ordered pairs (x, y) in which each x-coordinate is paired with only one y-coordinate. In a list of ordered pairs belonging to a function, no x-coordinate is repeated.

You can use a table to represent a function. Suppose you read a book at the constant rate of 50 pages an hour.

Elapsed Time (hours)	Pages Read
1	50
2	100
3	150
4	200
5	250
6	300

The number of pages you read can be described in terms of the number of hours you read.

In a functional relationship, for any given input there is a unique output.



Input an x-value and you get a y-value.

If you are given an x-value belonging to a function, you can find the corresponding y-value.

Do you see that . . .

If you input 5 hours into the function above, the output will be 250 pages.

There are two ways to test a set of ordered pairs to see whether it is a function.

Examine the list of ordered pairs.

If a set of ordered pairs is a function, no x-coordinate in the set is repeated. No x-coordinate should be listed with two different y-coordinates.

Is this set of ordered pairs a function?

$$\{(0, 4), (-2, 2), (0, 0)\}$$

Examine the set of ordered pairs.

- The number 0 is paired with 4; -2 is paired with 2; and 0 is paired with 0.
- Two ordered pairs, (0, 4) and (0, 0), have the same x-coordinate. In a functional relationship, no x-coordinate should repeat.

This set of ordered pairs is not a function.

Is this set of ordered pairs a function?

$$\{(5, -1), (-3, 4), (0, -1), (2, 7)\}$$

Examine the set of ordered pairs.

- The x-coordinates are 5, -3, 0, and 2. None of the x-coordinates repeat.
- There are two ordered pairs, $(5, -1)$ and $(0, -1)$, that have different x-coordinates but the same y-coordinate. This does not prevent this set of ordered pairs from being a functional relationship.

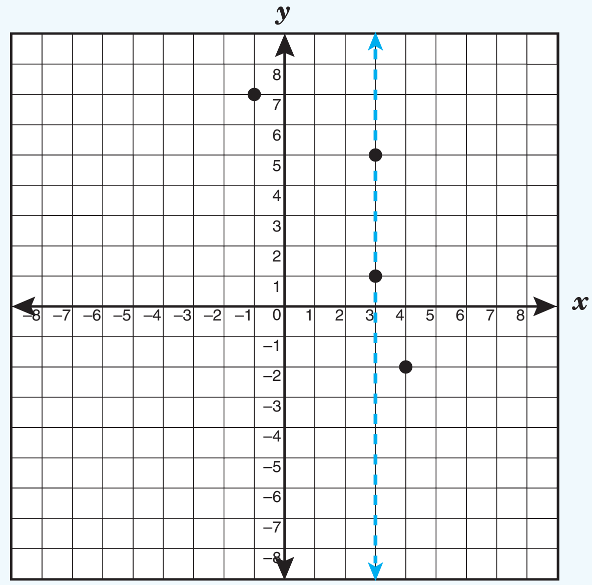
This set of ordered pairs is a function.

Do you see that . . .

Examine a graph of the function.

Use a vertical line to determine whether two points have the same x-coordinate. If two points in the function lie on the same vertical line, then they have the same x-coordinate, and the set of ordered pairs is not a function.

Do the ordered pairs graphed below represent a function?



The ordered pairs (3, 5) and (3, 1) lie on a common vertical line.

They have the same *x*-coordinate, 3, but different *y*-coordinates, 5 and 1.

This graph does not represent a function because two points lie on the same vertical line.

In a function, the *y*-coordinate is described in terms of the *x*-coordinate. The value of the *y*-coordinate depends on the value of the *x*-coordinate.

Do you see that . . .

Lynn collects figurines. At the end of every month, she goes to the hobby store to buy as many new figurines as she can afford. The figurines cost $2 each.

If Lynn has $6 available at the end of the month, she can buy 3 figurines.

If Lynn has $8 available at the end of the month, she can buy 4 figurines.

In this function, the amount of available money is the **independent** quantity. The number of figurines Lynn can buy is the **dependent** quantity.

You can describe the function using an equation. In the equation below, the number of figurines Lynn can buy in any given month, n, is given in terms of d, the amount of money in dollars she has to spend.

$$n = \frac{d}{2}$$

- The amount of money Lynn has to spend, d, determines how many figurines she can buy. So d is the independent variable.
- The number of figurines that Lynn can buy, n, depends on the amount of money she has to spend. So n, the number of figurines, is the dependent variable.
- The cost of each figurine remains constant at $2 each. In the equation $n = \frac{d}{2}$, the number 2 is a **constant** because it does not change.

Suppose the cost of attending college for a semester is $18 per credit hour plus a $75 student activity fee.

The equation $c = 18h + 75$ describes the cost of attending college, c, in terms of h, the number of credit hours taken.

In this function, h is the independent variable, and c is the dependent variable. The numbers 18 and 75 are constants.

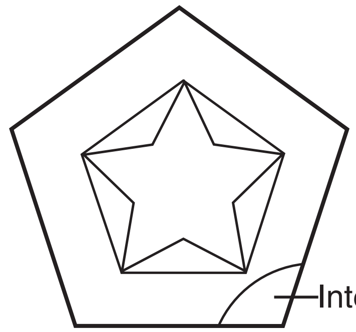
Carson works from 35 to 40 hours per week in a coffee shop. He makes $6.00 per hour plus an average of $25 per week in tips. The total amount of money in dollars that Carson makes each week, s, can be estimated with the equation $s = 6h + 25$, where h represents the number of hours he works.

In this equation, what is the dependent variable, and what are the constants?

- The amount of money Carson makes each week, s, depends on the number of hours he works, h. Therefore, s is the dependent variable.
- The constants are 6, his hourly wage, and 25, his average weekly tips, because these numbers do not change.

Try It

Julio builds toys in the shape of regular polygons. Some toys have 3 sides, but others are shaped like polygons with many sides. In order to cut out the regular polygons with his saw, Julio needs to know the size of the interior angle needed for each toy.



To find the size of the interior angle, Julio uses the function below, which shows the relationship between n, the number of sides a regular polygon has, and m, the measure of its interior angle.

$$m = \frac{180(n - 2)}{n}$$

In this functional relationship, which value is the dependent quantity, and which is the independent quantity?

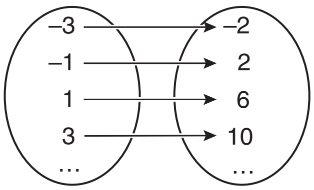
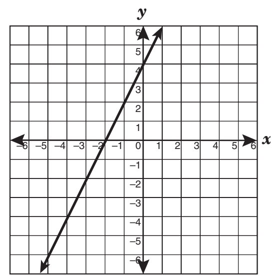
The ______________________ quantity is the number of sides in the polygon.

The ______________________ quantity is the measure of the interior angle because it depends on the number of sides in the polygon.

The independent quantity is the number of sides in the polygon. The dependent quantity is the measure of the interior angle because it depends on the number of sides in the polygon.

How Can You Represent a Function?

Functional relationships can be represented in a variety of ways.

<table>
<tr><th>Method</th><th>Description</th><th>Example</th></tr>
<tr><td>List</td><td>List the ordered pairs.</td><td>{(−3, −2), (−1, 2), (1, 6), (3, 10), …}</td></tr>
<tr><td>Table</td><td>Place the ordered pairs in a table.</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−3</td><td>−2</td></tr><tr><td>−1</td><td>2</td></tr><tr><td>1</td><td>6</td></tr><tr><td>3</td><td>10</td></tr><tr><td>…</td><td>…</td></tr></table></td></tr>
<tr><td>Mapping</td><td>Draw a picture that shows how the ordered pairs are formed.</td><td></td></tr>
<tr><td>Description</td><td>Use words to describe the functional relationship.</td><td>The y-values for a set of points are 4 more than twice the corresponding x-values.</td></tr>
<tr><td>Equation</td><td>Write an equation that describes the y-coordinate in terms of the x-coordinate.</td><td>$y = 2x + 4$</td></tr>
<tr><td>Function notation</td><td>Write a special type of equation that uses f(x) to represent y.</td><td>$f(x) = 2x + 4$</td></tr>
<tr><td>Graph</td><td>Graph the ordered pairs.</td><td></td></tr>
</table>

To use **function notation** to describe a function, give the function a name, typically a letter such as f, g, or h. Then use an algebraic expression to describe the y-coordinate of an ordered pair.

Do you see that . . .

Suppose $f(x) = 2x + 1.5$.

- This function is read as "f of x equals 2 times x plus 1.5."
- If you input x, the output will be $2x + 1.5$.
- This means that the y-coordinate of an ordered pair is $2x + 1.5$.

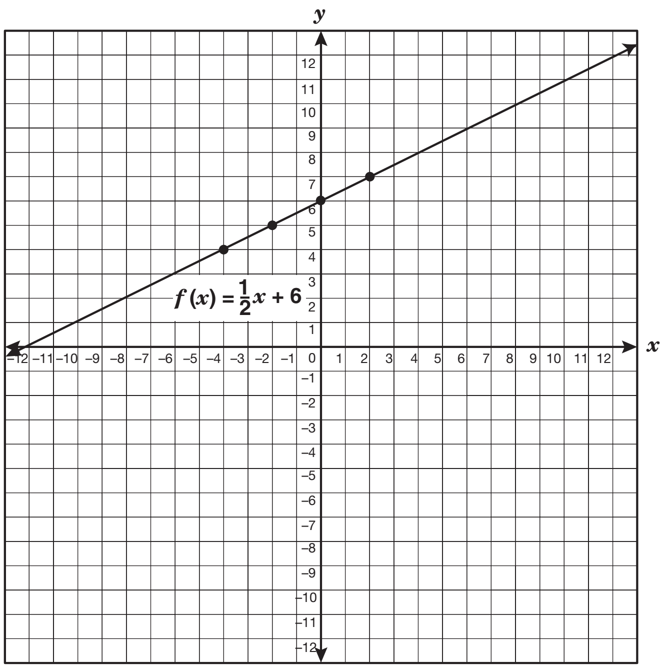
The function described by $f(x) = 2x + 1.5$ is the same as the function described by $y = 2x + 1.5$. In this function, an ordered pair looks like this: $(x, 2x + 1.5)$.

Here are three methods you can use to determine whether two different representations of a function are equivalent.

Method	Action
Match a list or table of ordered pairs to a graph.	• Show that each ordered pair listed matches a point on the graph.
Match an equation to a graph.	• Determine whether they are both linear or quadratic functions. • Find points on the graph and show that their coordinates satisfy the equation. • Find points that satisfy the equation and show that they are on the graph.
Match a verbal description to a graph, an equation, or an expression written in function notation.	• Use the verbal description to find ordered pairs belonging to the function and then show that they satisfy the graph, equation, or function rule. • Find points on the graph or ordered pairs satisfying the equation or rule and show that they satisfy the verbal description.

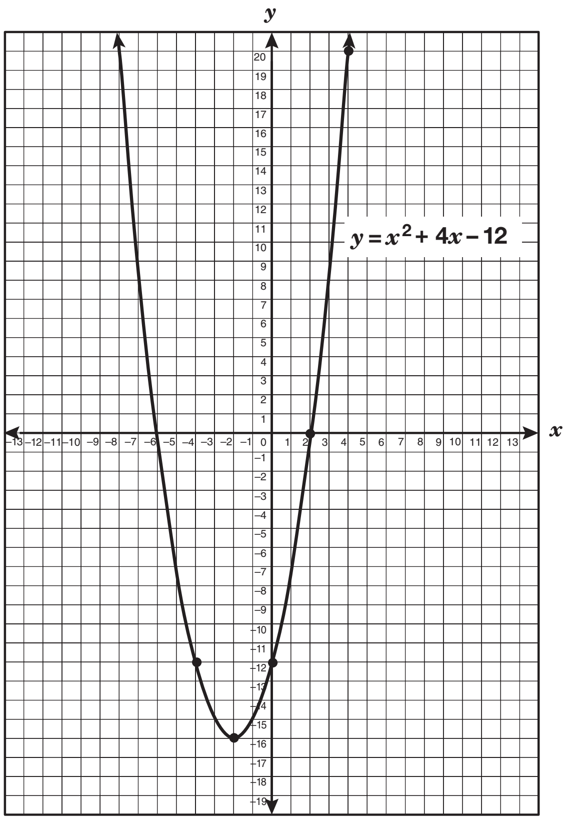
Any equation of the form $y = mx + b$ is a **linear function**. Its graph will be a line.

x	y
−4	4
−2	5
0	6
2	7



Any equation of the form $y = ax^2 + bx + c$ is a **quadratic function**. Its graph will be a parabola.

x	y
−4	−12
−2	−16
0	−12
2	0
4	20



Does the ordered pair (8, 48) belong to a function in which the *y*-coordinate in each ordered pair is always 7 times the *x*-coordinate?

- Try to match the ordered pair to the verbal description. The description says that *y* equals 7 times *x*. If the *x*-coordinate is 8, then the *y*-coordinate should be $7 \cdot 8 = 56$, not 48 as given.

The ordered pair (8, 48) does not belong to this function.

What ordered pairs belong to this function?

- Use the rule for the function to find ordered pairs that belong to the function. For example, if the *x*-coordinate in an ordered pair belonging to this function is 6, then the *y*-coordinate would be $7 \cdot 6 = 42$.

The ordered pair (6, 42) belongs to this function.

A variety of methods of representing a function are shown below. Which of these examples represents a function that is different from the other functions?

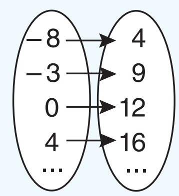
A. Verbal Description
The value of *y* is 12 more than the value of *x*.

B. List of Selected Values
{(3, 15), (−2, 10), (1, 13), (0, 12), (−5, 7), …}

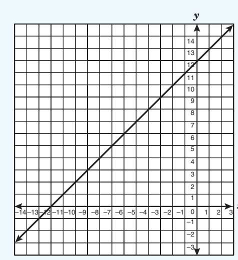
C. Table of Selected Values

x	*y*
5	60
−3	−36
1	12
0	0
…	…

D. Mapping of Selected Values



E. Graph



F. Equation
$y = x + 12$

Look at the ordered pairs that make up each function.

- In Examples A, B, D, E, and F, each number is paired with a number 12 more than itself.
- In Example C, for each ordered pair listed, the *y*-coordinate is 12 times the *x*-coordinate, not 12 more than the *x*-coordinate. So Example C includes ordered pairs that are not in the other examples.

Only Example C represents a different function.

The table below presents selected values in a functional relationship.

x	-3	-1	2	4
y	-7	-5	-2	0

Write an equation that describes this functional relationship.

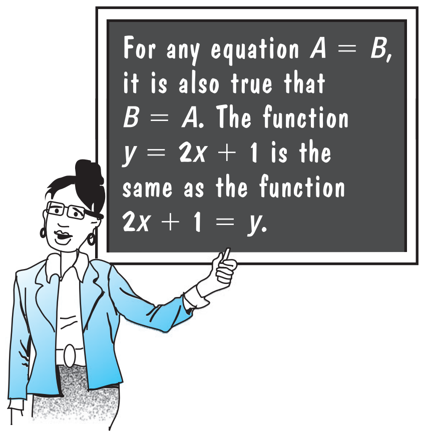
- Look for a pattern between the ordered pairs that belong to the function.

  The y-coordinate appears to be 4 less than the x-coordinate, so $x - 4 = y$ should represent the pattern.
- Check this equation for each pair.

	$\underline{x - 4 = y}$
For $x = -3$	$-3 - 4 = -7$
For $x = -1$	$-1 - 4 = -5$
For $x = 2$	$2 - 4 = -2$
For $x = 4$	$4 - 4 = 0$

  The y-coordinate in each number pair is 4 less than the x-coordinate.

The rule for this function can be represented by the equation $x - 4 = y$ or by the equation $y = x - 4$.



The table on the right presents selected values in a functional relationship.

x	y
1	2
3	8
-5	-16
15	44

Using function notation, write a rule that represents the relationship.

- Look for a pattern between the ordered pairs that belong to the function.

  The y-coordinate appears to be 3 times the x-coordinate minus 1.
- Check this pattern for each pair.

	$\underline{3x - 1 = y}$
$(1, 2)$	$3 \cdot 1 - 1 = 2$
$(3, 8)$	$3 \cdot 3 - 1 = 8$
$(-5, -16)$	$3 \cdot -5 - 1 = -16$
$(15, 44)$	$3 \cdot 15 - 1 = 44$

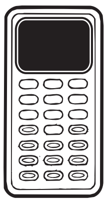
  The y-coordinate is equal to 3 times the x-coordinate minus 1.

The rule for this function can be represented by the equation $y = 3x - 1$.

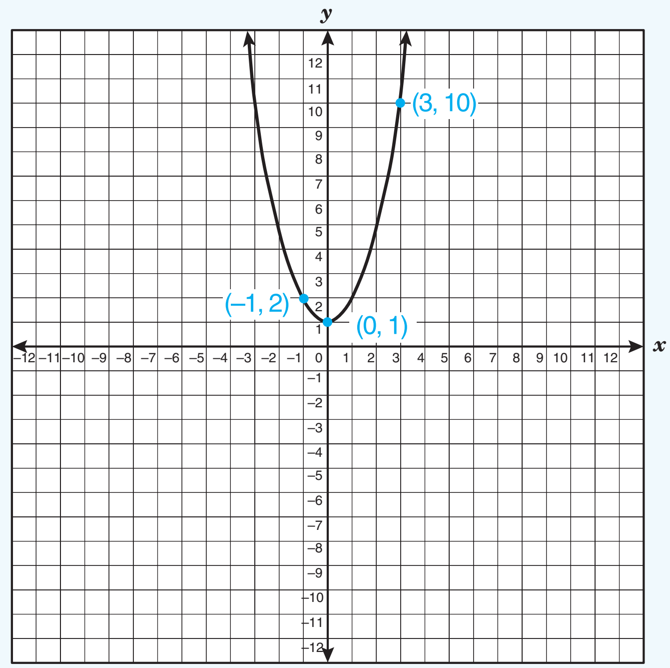
Replace y with $f(x)$ to express the rule in function notation: $f(x) = 3x - 1$.

Do you see that . . .

MATHEMATICS



Does the graph below represent the same function as the equation $y = x^2 + 1$?



- The function $y = x^2 + 1$ is a quadratic function because there is an x^2 term in its equation. Its graph should be a parabola. The graph above is a parabola, so the equation and the graph are the same type of function.
- Show that the equation and the graph represent the same quadratic function. You must find at least three points in common.
- First check the coordinates of points on the graph to see whether they satisfy the equation. Pick points on the graph whose coordinates are easy to read. For example, you might choose (0, 1), (3, 10), and (−1, 2).

Point (0, 1)	Point (3, 10)	Point (−1, 2)
$x = 0$ and $y = 1$	$x = 3$ and $y = 10$	$x = -1$ and $y = 2$
$y = x^2 + 1$	$y = x^2 + 1$	$y = x^2 + 1$
$1 \stackrel{?}{=} 0^2 + 1$	$10 \stackrel{?}{=} 3^2 + 1$	$2 \stackrel{?}{=} (-1)^2 + 1$
$1 \stackrel{?}{=} 0 + 1$	$10 \stackrel{?}{=} 9 + 1$	$2 \stackrel{?}{=} 1 + 1$
$1 = 1$	$10 = 10$	$2 = 2$

The points (0, 1), (3, 10), and (−1, 2) are points on the graph, and their coordinates satisfy the equation.

- Next find ordered pairs of numbers that satisfy the equation and confirm that the points are on the graph. Pick values that are easy to substitute, like $x = 1$, $x = 2$, or $x = -2$, and find the corresponding values for y.

$\underline{x = 1}$	$\underline{x = 2}$	$\underline{x = -2}$
$y = x^2 + 1$	$y = x^2 + 1$	$y = x^2 + 1$
$y = (1)^2 + 1$	$y = (2)^2 + 1$	$y = (-2)^2 + 1$
$y = 1 + 1$	$y = 4 + 1$	$y = 4 + 1$
$y = 2$	$y = 5$	$y = 5$

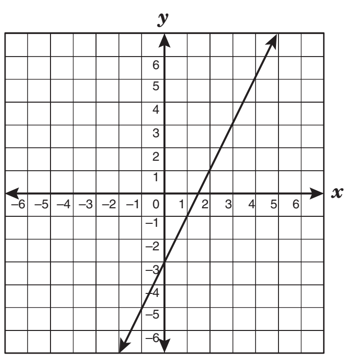
  The ordered pairs (1, 2), (2, 5), and (−2, 5) satisfy the equation.

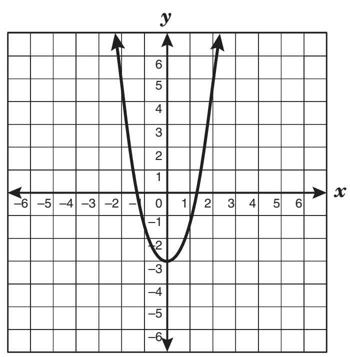
  Confirm that these ordered pairs are points on the graph. Look at the graph. Yes, all three points are on the graph.

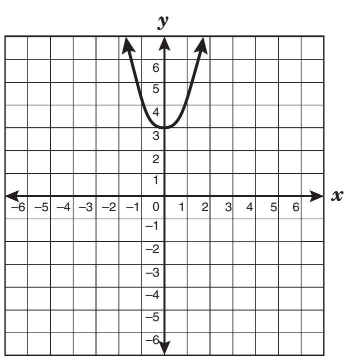
The graph does represent the relationship $y = x^2 + 1$.

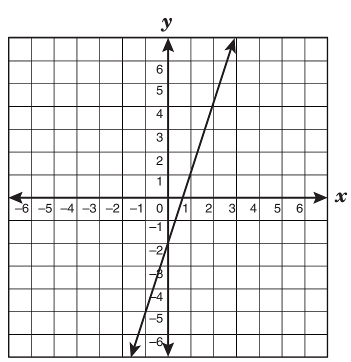
Try It

The equation $y = 2x^2 - 3$ represents a functional relationship. Which graph represents this function?

A


C


B


D


Determine whether the equation is a linear or quadratic function.

It is a ____________________ function because it contains the term x^2.

Its graph must be a ____________________ .

Answer choices ______ and ______ cannot be the graph of this function because they are ______________ .

Determine which parabola is the correct graph.

See if the point (0, 3) in answer choice B satisfies the equation $y = 2x^2 - 3$.

When $x = 0$ and $y =$ ______, is the equation $y = 2x^2 - 3$ true?

Does $3 = 2(\_\_\_\_\_\_)^2 -$ ______ ?

No, ______ $\neq$ ______ .

Answer choice B is ________________________ .

See if the point $(0, -3)$ in answer choice C satisfies the equation $y = 2x^2 - 3$.

When $x = 0$ and $y =$ ______ , is the equation $y = 2x^2 - 3$ true?

Does ______ $= 2(\_\_\_\_\_\_)^2 -$ ______ ?

Yes, ______ $=$ ______ .

Answer choice C is _____________________ .

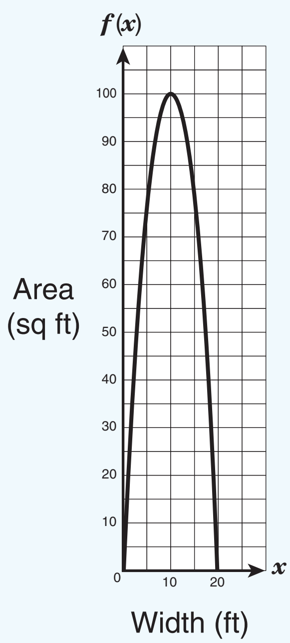
It is a quadratic function because it contains the term x^2. Its graph must be a parabola. Answer choices A and D cannot be the graph of this function because they are lines. Determine which parabola is the correct graph. See if the point (0, 3) in answer choice B satisfies the equation $y = 2x^2 - 3$. When $x = 0$ and $y = 3$, is the equation $y = 2x^2 - 3$ true? Does $3 = 2(0)^2 - 3$? No, $3 \neq -3$. Answer choice B is not correct. See if the point $(0, -3)$ in answer choice C satisfies the equation $y = 2x^2 - 3$. When $x = 0$ and $y = -3$, is the equation $y = 2x^2 - 3$ true? Does $-3 = 2(0)^2 - 3$? Yes, $-3 = -3$. Answer choice C is correct.

MATHEMATICS

How Can You Draw Conclusions from a Functional Relationship?

Use these guidelines when interpreting functional relationships in a real-life problem.

- Understand the problem.
- Identify the quantities involved and any relationships between them.
- Determine what the variables in the problem represent.
- For graphs: Determine what quantity each axis on the graph represents. Look at the scale that is used on each axis.
- For tables: Determine what quantity each column in the table represents.
- Look for trends in the data. Look for maximum and minimum values in graphs.
- Look for any unusual data. For example, does a graph start at a nonzero value? Is one of the problem's variables negative at any point?
- Match the data to the equations or formulas in the problem.



Daisy wants to make a rectangular flower garden. She needs to put a fence around the garden so her dog will not dig in it. Daisy uses a total of 40 feet of fencing. She can choose different combinations of length and width for her garden.

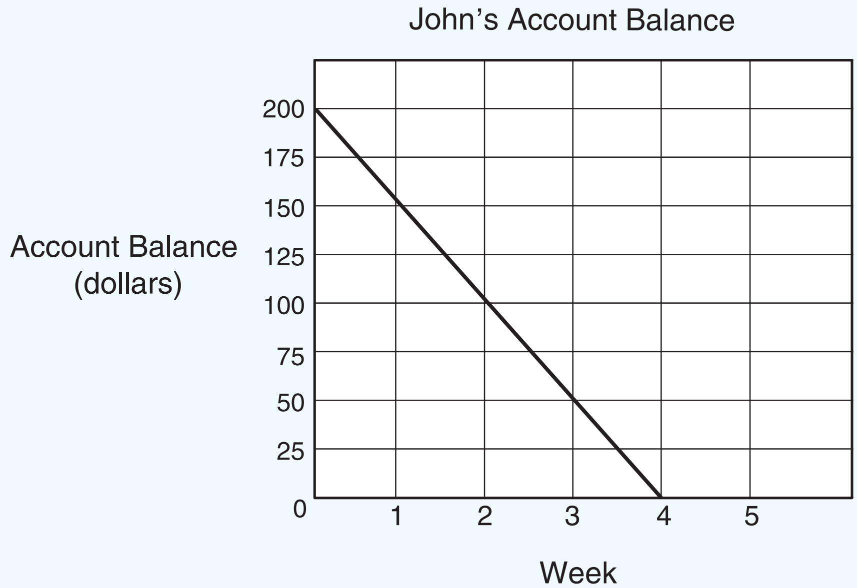
The graph shows the relationship between the number of feet of fencing Daisy can use for the width of her garden, x, and the area of the garden, $f(x)$. Use the graph to determine what width will give the greatest area for the garden.

- Notice that as the width of the rectangle increases, the area increases until $x = 10$. Then the area begins to decrease.
- The function appears to achieve a maximum value at $x = 10$.

Using only 40 feet of fencing, a width of 10 feet will give the greatest area for the garden.

John has $200 in his savings account. Every week for four consecutive weeks, he goes to the bank and withdraws half of the money in his account.

Does the graph below represent John's account balance accurately?



- The graph starts at the point (0, 200), the beginning of the time period and the point at which John has a balance of $200 in his account.
- Subsequent points on the graph indicate a decline in John's account balance, so the graph reflects withdrawals from the account. But are the data on the graph consistent with the information given in the problem?
- Check the data on the graph against the problem, which says that every week John withdraws half of the money in his account.

$$\text{Week 1: } \frac{1}{2}(200) = 100$$

$$\text{Week 2: } \frac{1}{2}(100) = 50$$

$$\text{Week 3: } \frac{1}{2}(50) = 25$$

$$\text{Week 4: } \frac{1}{2}(25) = 12.50$$

The data in the graph do not match the description given in the problem. For example, the graph shows a balance of $0 for Week 4 when the balance is actually $12.50.

Now practice what you've learned.

Question 1

On average, Jay can ride his bike 12 miles in one hour. The function $m = 12h$ represents the number of miles, m, he can ride in h hours. Which quantity in this relationship is the dependent quantity?

A The number of miles he rides

B The number of hours he rides

C The miles per hour he rides

D Not Here

Answer Key: page 288

Question 2

Zachary is making an isosceles triangle to use as a model in math class. Its perimeter will be 24 inches. The equation $b = 24 - 2s$ describes b, the length of the triangle's third side, in terms of s, the length of each of its two congruent sides. What is the dependent variable in this equation?

A b

B s

C 24

D 2

Answer Key: page 288

Question 3

Which of the following tables does not represent a functional relationship?

A

x	y
1	−9
2	−5
−1	9
−2	5

B

x	y
1	9
1	−9
2	5
2	−5

C

x	y
1	9
−1	−9
2	9
−2	−9

D

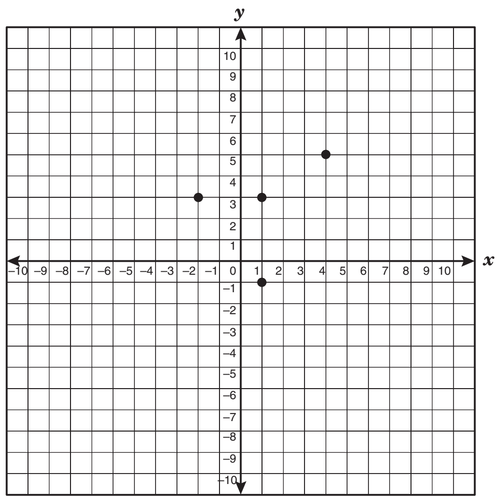
x	y
1	9
−1	5
2	−9
−2	−5

Answer Key: page 288

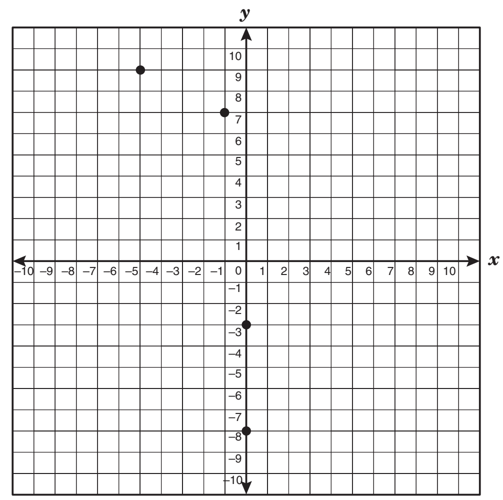
Question 4

Which of the following graphs represents a functional relationship?

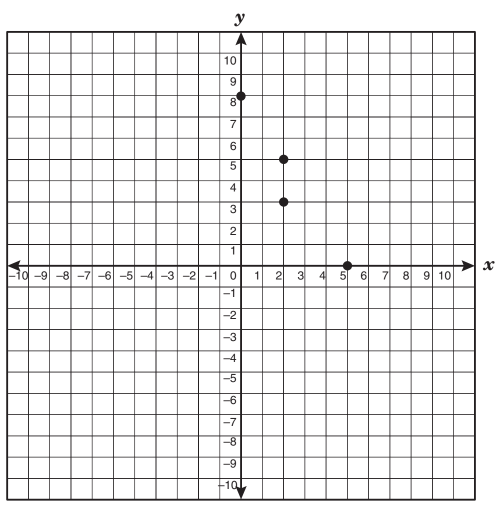
A



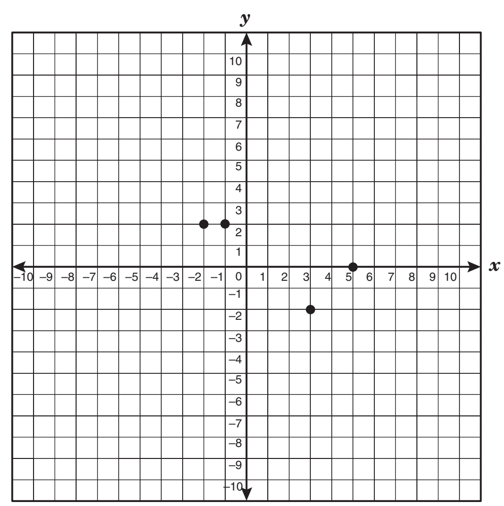
C



B



D



Answer Key: page 288

Question 5

Each month Jean's phone bill includes a $25 basic fee plus a charge of $.07 per minute for the number of minutes of long-distance calls she makes. Which equation best describes the total amount of Jean's monthly phone bill, t, in terms of m, the number of minutes of long-distance calls she makes?

A $t = 0.07 + 25m$

B $t = 25 + 0.07m$

C $t = 25 \cdot 0.07m$

D $t = 25 \cdot 7m$

Answer Key: page 288

Question 6

Rupert is driving 182 miles to Houston for a convention. He has already driven x miles of the trip. If Rupert drives below 70 miles per hour for the remainder of the trip, which inequality best represents the amount of time in hours, t, that it will take him to complete the remainder of his drive to Houston?

A $t < \dfrac{182 - x}{70}$

B $t > \dfrac{70}{182 - x}$

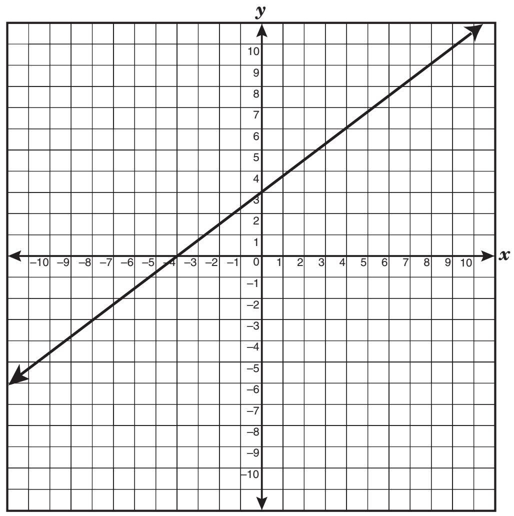
C $t < \dfrac{70}{182 - x}$

D $t > \dfrac{182 - x}{70}$

Answer Key: page 288

Question 7

Which of the following equations represents the same function as the graph below?



A $y = \dfrac{3}{4}x + 3$

B $y = \dfrac{4}{3}x + 3$

C $y = \dfrac{3}{4}x - 4$

D $y = \dfrac{4}{3}x - 4$

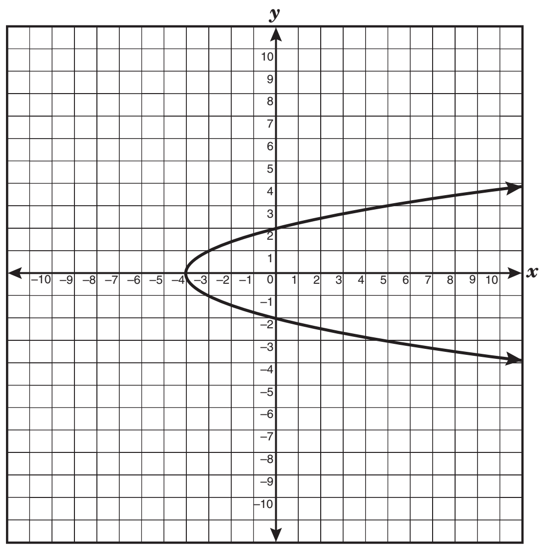
Answer Key: page 288

MATHEMATICS

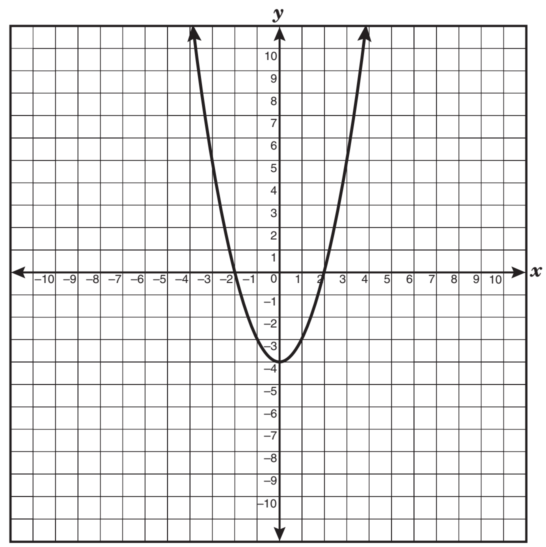
Question 8

Which graph best represents the function $f(x) = x^2 - 4$?

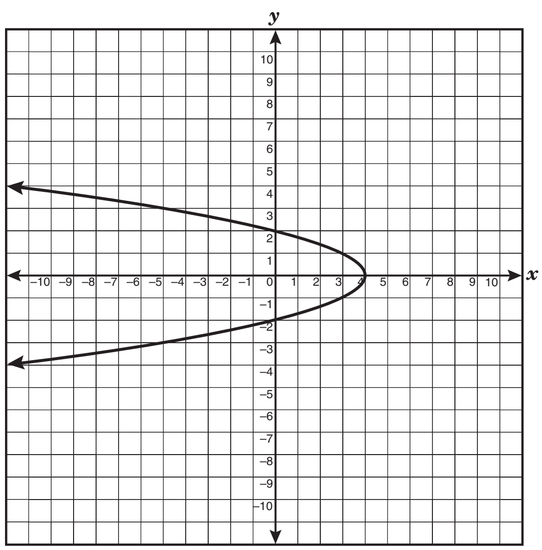
A



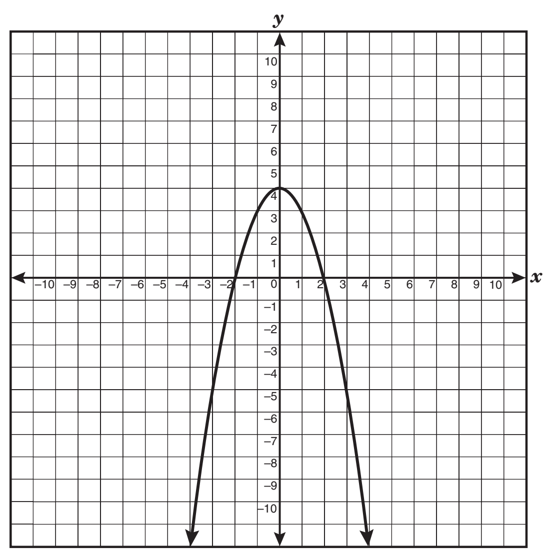
C



B



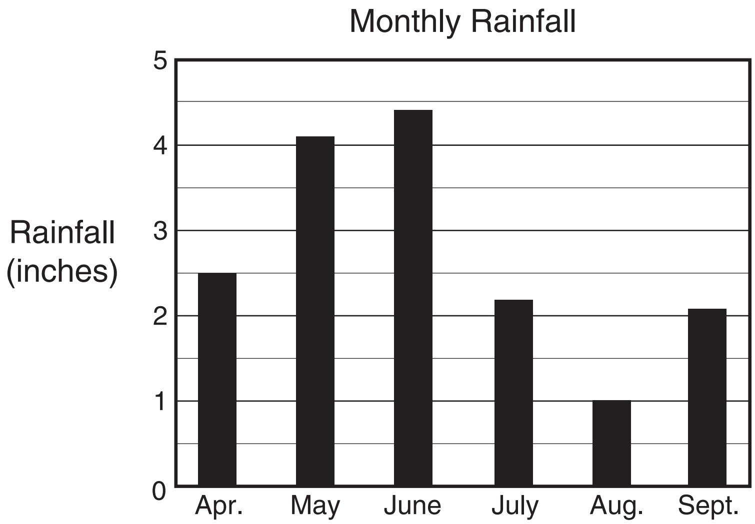
D



Answer Key: page 289

Question 9

For each month from April through September, Ralph recorded the total number of inches of rainfall. The graph below shows the monthly total rainfall he recorded.



Based on the information in this graph, which of the following conclusions is not reasonable?

A More rain fell in April than in September.

B August was the month with the least rainfall.

C More rain fell in June than in April and July combined.

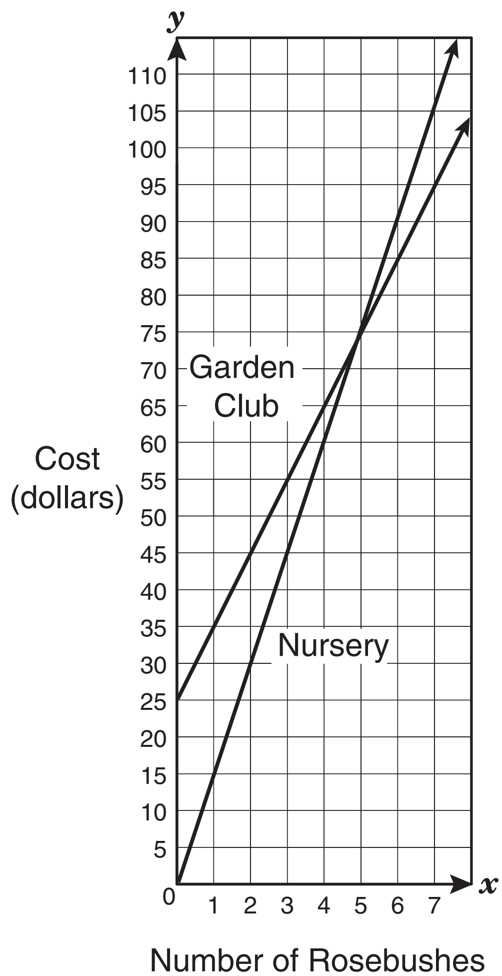
D From 1 to 4.4 inches of rain fell in each of the months recorded.

Answer Key: page 289

MATHEMATICS

Question 10

Larry is considering joining the Garden Club. If he pays a $25 membership fee, he can buy rosebushes from the club at a reduced price of $10 each. If he does not join the club, he can buy rosebushes from a local nursery for $15 each. The graph below compares the cost of buying rosebushes from the Garden Club and from the local nursery.



How many rosebushes will Larry have to buy from the Garden Club before he would begin to save money?

A 25

B 5

C 75

D 7

Answer Key: page 289

Objective 2

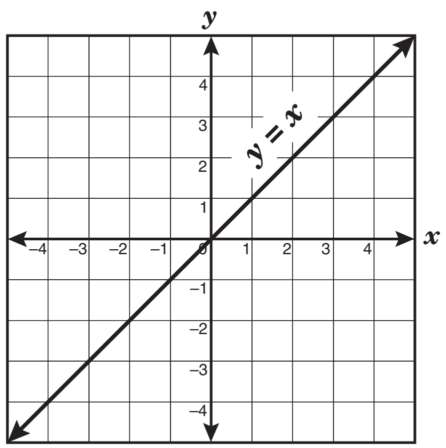
The student will demonstrate an understanding of the properties and attributes of functions.

For this objective you should be able to

- use the properties and attributes of functions;
- use algebra to express generalizations and use symbols to represent situations; and
- manipulate symbols to solve problems and use algebraic skills to simplify algebraic expressions and solve equations and inequalities in problem situations.

What Are Parent Functions?

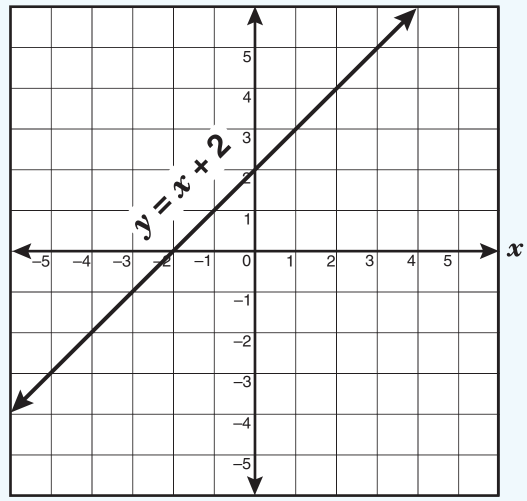
The simplest linear function, $y = x$, is the **linear parent function**.



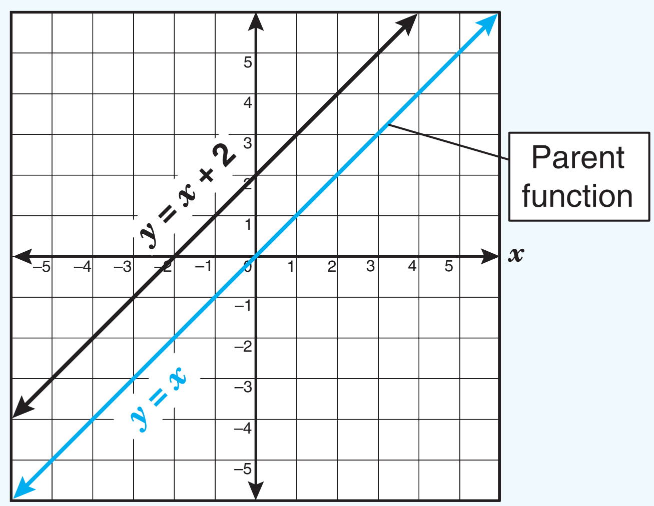
- If the graph of any function is a line, then its parent function is $y = x$.
- A linear equation never has variables raised to a power other than 1.
- If an equation is linear, then its parent function is $y = x$.

MATHEMATICS

What is the parent function of this graph?

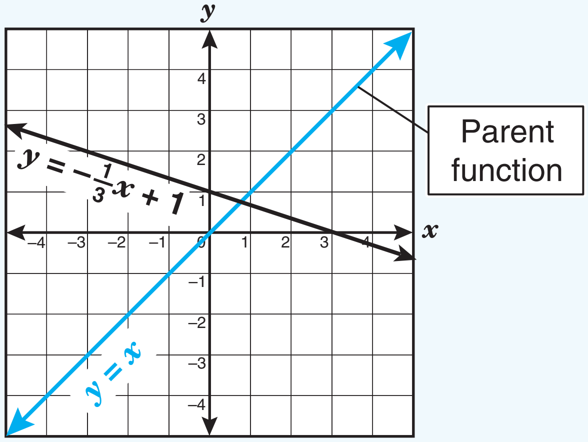


Since the graph of $y = x + 2$ is a line, its parent function is the linear parent function, $y = x$.

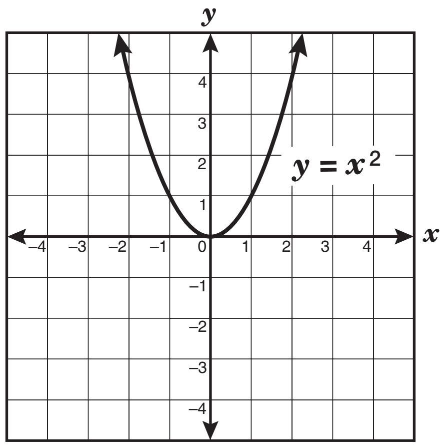


What is the parent function of the equation $y = -\frac{1}{3}x + 1$?

Since the equation $y = -\frac{1}{3}x + 1$ is a linear equation, its graph is a line. Its parent function is the linear parent function, $y = x$.

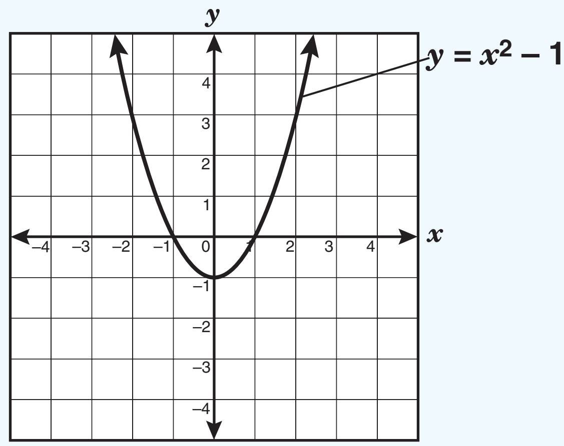


The simplest quadratic function, $y = x^2$, is the **quadratic parent function**.

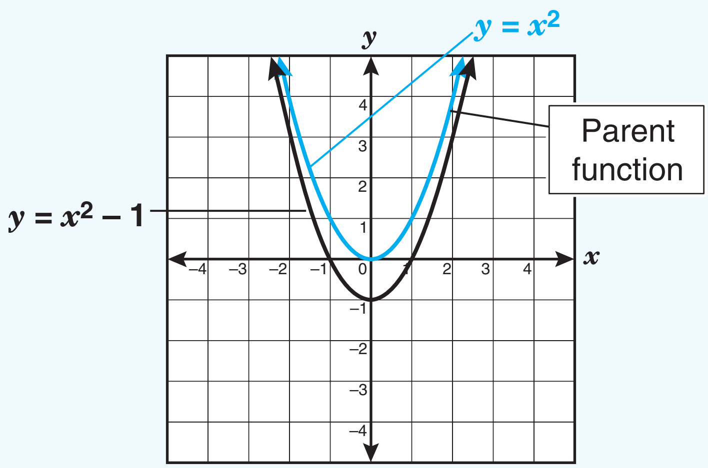


- If the graph of any function is a parabola, then its parent function is $y = x^2$.
- If an equation can be written in the form $y = ax^2 + bx + c$, then it is quadratic.
- If an equation can be written in this form, then its parent function is $y = x^2$.

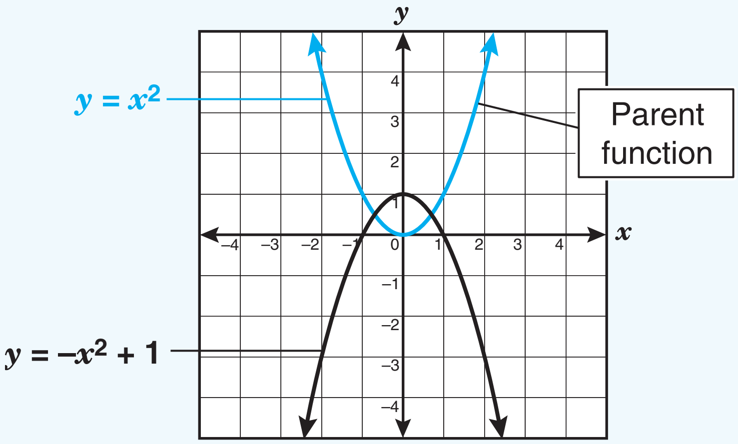
What is the parent function of this graph?



Since the graph of $y = x^2 - 1$ is a parabola, its parent function is the quadratic parent function, $y = x^2$.



What is the parent function of $y = -x^2 + 1$?



The equation $y = -x^2 + 1$ is a quadratic equation; therefore, its parent function is the quadratic parent function, $y = x^2$.

What Are the Domain and Range of a Function?

A **function** is a set of ordered pairs of numbers (x, y) such that no x-values are repeated. The domain and range of a function are sets that describe those ordered pairs.

	Definition	Example {(0,1), (2, 6), (3, 5)}
Domain	All the x-coordinates in the function's ordered pairs	{0, 2, 3}
Range	All the y-coordinates in the function's ordered pairs	{1, 5, 6}

- The **domain** is the set of all the values of the independent variable, the x-coordinate.
- The **range** is the set of all the values of the dependent variable, the y-coordinate.

See Objective 1, page 92, for more information about functions.

Identify the domain and range of the function below.

{(2, 7), (4, 11), (6, 15), (8, 19)}

The domain is the set of x-coordinates in the ordered pairs: {($\underline{2}$, 7), ($\underline{4}$, 11), ($\underline{6}$, 15), ($\underline{8}$, 19)}. The domain is {2, 4, 6, 8}.

The range is the set of y-coordinates in the ordered pairs: {(2, $\underline{7}$), (4, $\underline{11}$), (6, $\underline{15}$), (8, $\underline{19}$)}. The range is {7, 11, 15, 19}.

Try It

What are the domain and range of the function below?

$$\{(1, -9), (2, -16), (3, -25), (4, -36), (5, -49)\}$$

The domain of a function is the set of all ______-coordinates.

The domain of this function is

{______, ______, ______, ______, ______}.

The range of a function is the set of all ______-coordinates.

The range of this function is {______, ______, ______, ______, ______}.

The domain of a function is the set of all x-coordinates. The domain of this function is {1, 2, 3, 4, 5}. The range of a function is the set of all y-coordinates. The range of this function is {–49, –36, –25, –16, –9}.

The domain and range of algebraic functions are usually assumed to be the set of all real numbers. In some cases, however, the domain or range of a function may be a subset of the real numbers because certain numbers would not make sense in a real-life problem situation.

The number of shoes in n pairs of shoes can be expressed by the function $s = 2n$. Are there any values that would not be reasonable to include in the domain or range of this function?

- The domain of this function is the set of values you may choose for n, the independent variable. Would it be reasonable to let $n = -2$? No. The variable n represents a number of pairs of shoes, so it must be a nonnegative integer. The domain is the set of nonnegative integers, $\{0, 1, 2, 3, \ldots\}$.

It would not be reasonable to include any other numbers in the domain.

- The range of this function is the set of values you will obtain for the dependent variable, s, the number of shoes in n pairs of shoes. Is it possible to get 5 as a value for s? Could a set of pairs of shoes have 5 shoes? No, 5 is not a reasonable value for the range of this function. Since 1 pair of shoes has 2 shoes, 2 pairs of shoes have 4 shoes, and so on, the range of this function is the set of multiples of 2, or $\{0, 2, 4, 6, \ldots\}$.

It would not be reasonable to include any other numbers in the range.

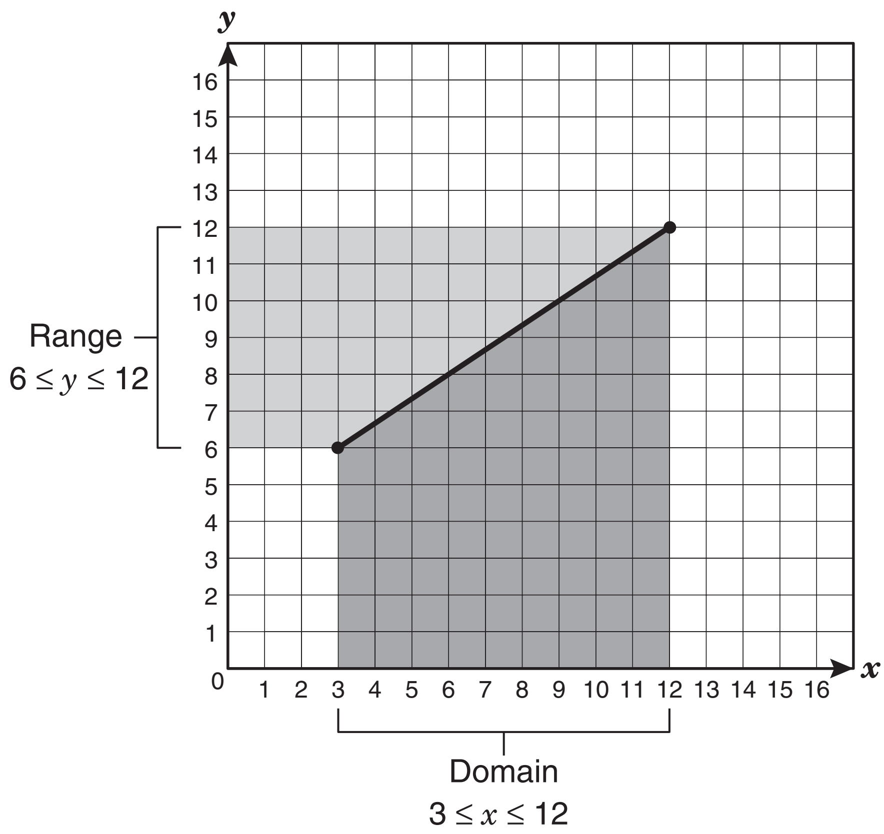
An electronics store estimates that its monthly return (profit or loss), r, can be described as a function of its total sales, s, for the month using the function $r = 0.15s - 250$. What set of numbers would be an appropriate domain and range for this function?

The domain of this function is the number of dollars the store has in sales for the month. This can be any nonnegative real number, since sales cannot be negative. Therefore, the domain of this function is $s \geq 0$.

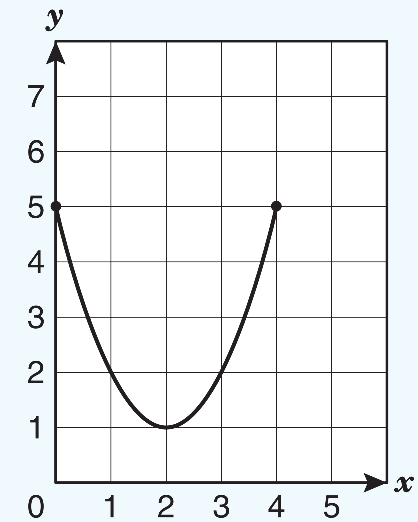
The range for this function is the number of dollars the store has in returns for the month. The minimum return would occur with a sales volume of $0. If $s = 0$, then $r = 0.15(0) - 250 = -250$. The store would show a loss of $250. Any sales greater than $0 would result in a greater return. Therefore, the range of this function is $r \geq -250$.

The graph of a function also can tell you about its domain and range.

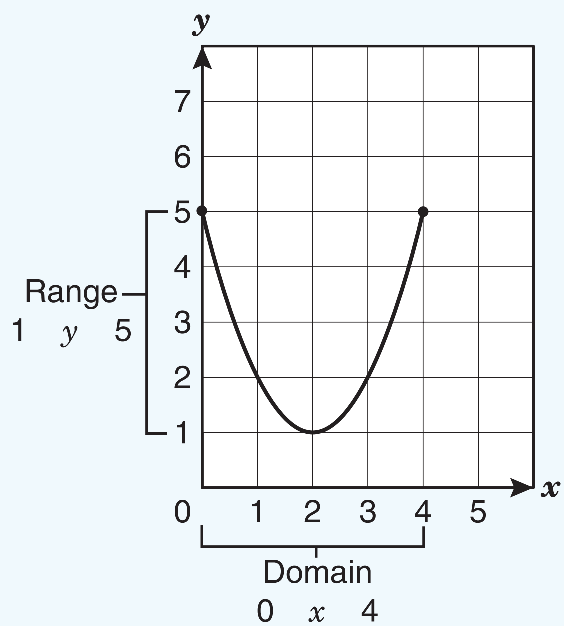
- The domain of a function is the set of all the x-coordinates in the function's graph.
- The range of a function is the set of all the y-coordinates in the function's graph.



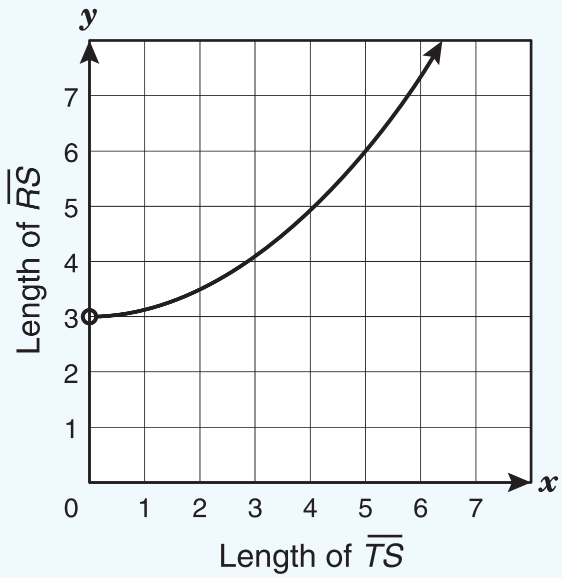
What inequalities best describe the domain and range of the function represented in this graph?



- The domain of this function is the set of *x*-values in the graph.
- The range of this function is the set of *y*-values in the graph.



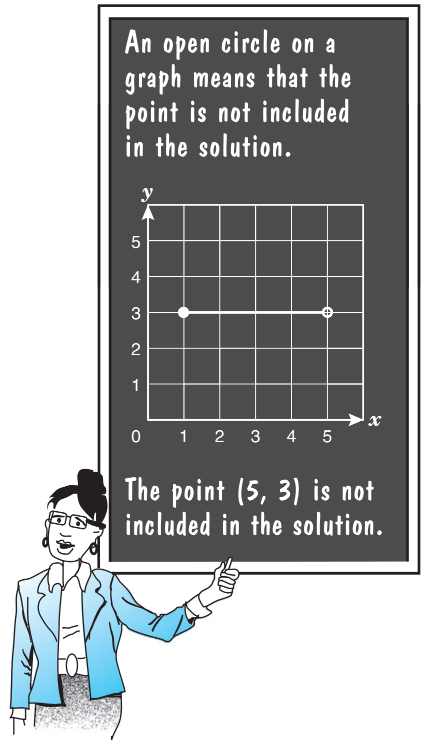
In right triangle RST, $\overline{RT}$ is 3 centimeters long. The graph below shows the relationship between the length of the other leg, $\overline{TS}$, and the length of the hypotenuse, $\overline{RS}$, of the triangle.



What are a reasonable domain and range for this function?

The domain of the function is the set of all the x-coordinates in the function. The length of a line segment must be positive, so the domain of the graph is $TS > 0$ centimeters.

The range of the function is the set of all the y-coordinates in the function. The smallest possible hypotenuse is when TS is at its minimum, just to the right of the point (0, 3). All other lengths for the hypotenuse must be greater than 3. So the range of the graph is $RS > 3$ centimeters.



MATHEMATICS

Try It

The average daily high temperature for the month of May in the town of Jonesville, Texas, is approximated by the function $t = 0.2n + 80$, where n is the date of the month. May has 31 days. What is a reasonable estimate of the range of this function?

The range of the function is the set of all the possible values for the ____________ variable in the function, the average daily high temperature.

To determine the range for the function, determine the minimum and ____________ daily high temperatures for the month.

The minimum daily high temperature will occur when $n = 1$.

$$t = 0.2n + 80$$
$$t = 0.2(\_\_\_\_) + 80$$
$$t = \_\_\_\_ + 80$$
$$t = \_\_\_\_$$

The maximum daily high temperature will occur when $n = 31$.

$$t = 0.2n + 80$$
$$t = 0.2(\_\_\_\_) + 80$$
$$t = \_\_\_\_ + 80$$
$$t = \_\_\_\_$$

A reasonable range for this function is ______ $\leq t \leq$ ______.

The range of the function is the set of all the possible values for the dependent variable in the function, the average daily high temperature. To determine the range for the function, determine the minimum and maximum daily high temperatures for the month.

The minimum daily high temperature will occur when $n = 1$.

$$t = 0.2n + 80$$
$$t = 0.2(1) + 80$$
$$t = 0.2 + 80$$
$$t = 80.2$$

The maximum daily high temperature will occur when $n = 31$.

$$t = 0.2n + 80$$
$$t = 0.2(31) + 80$$
$$t = 6.2 + 80$$
$$t = 86.2$$

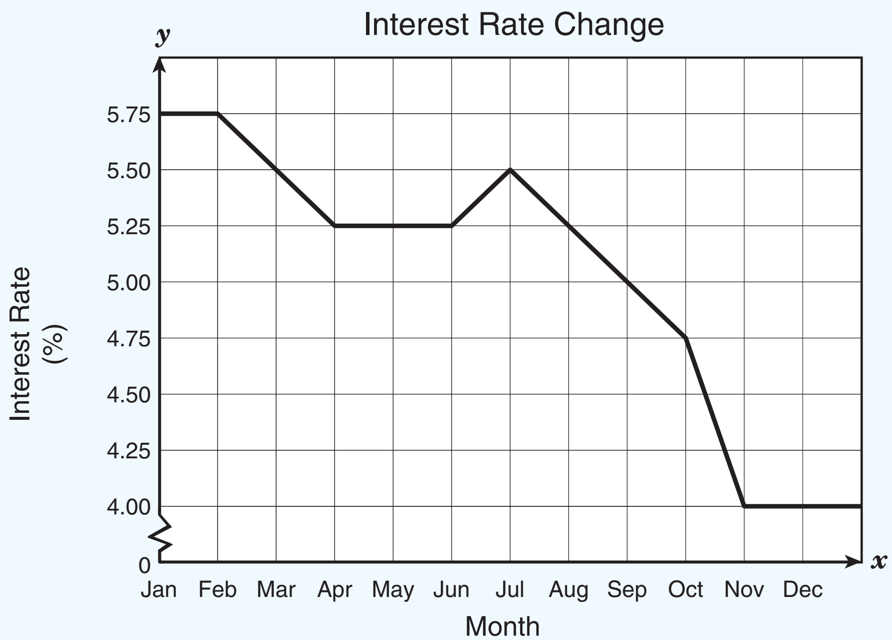
A reasonable range for this function is $80.2 \leq t \leq 86.2$.

How Can You Interpret a Problem from a Graph?

To interpret a problem situation described in terms of a graph, follow these guidelines.

- Identify the quantities that are being compared.
- Understand what relationship the graph is describing.
- Look at the scales used on the axes of the graph.
- Identify the domain or range of the function graphed.
- Look for patterns in the data—increases, decreases, or data that remain constant.

The graph below represents the interest rate a local bank offered to its savings-account customers over a 1-year period.



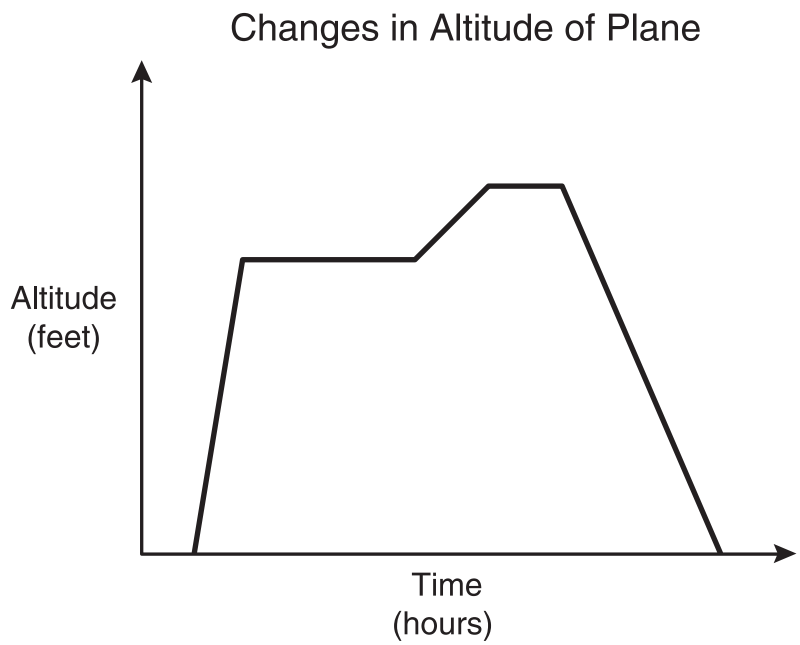
Describe the history of the interest rate offered and identify during which month the interest rate fell at the greatest rate.

The y-axis represents the interest rate offered. At the beginning of the year, the highest interest rate, 5.75%, was offered. It remained constant for 1 month, fell for the next 2 months, and then remained constant again for 2 months. Following that, the interest rate rose for 1 month but then fell steadily for the next 3 months. Then during the month of October, it fell at the greatest rate, dropping from 4.75% to 4.00%. This was a drop of 0.75% in one month. It remained constant for the last two months of the year.

MATHEMATICS

Try It

A plane flies from Dallas to Orlando. The graph below represents the plane's altitude.



Describe the flight of the plane in terms of its altitude during the trip from Dallas to Orlando.

The plane is on the ground, at a height of _____ feet, at the beginning of the flight.

The plane takes off and __________ altitude quickly.

It then flies at the __________ altitude for a while.

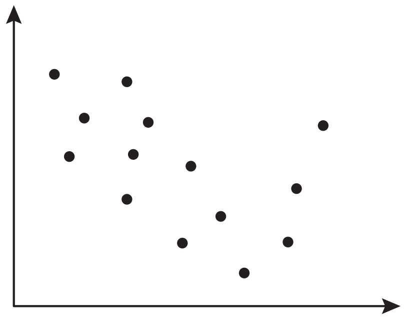
The plane climbs to a __________ altitude and flies at that height for a shorter time.

The plane quickly __________ and then lands.

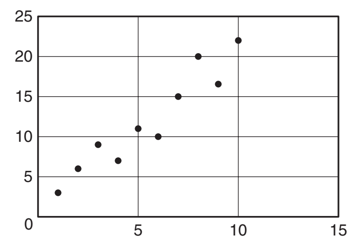
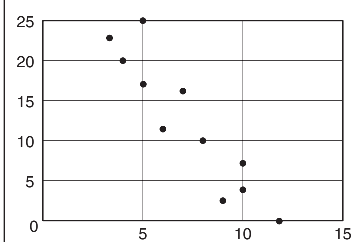
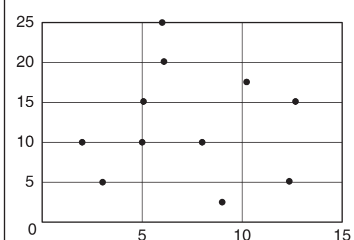
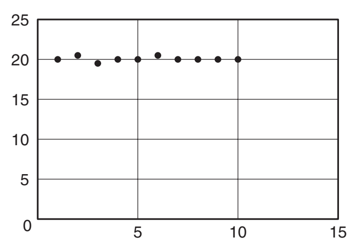
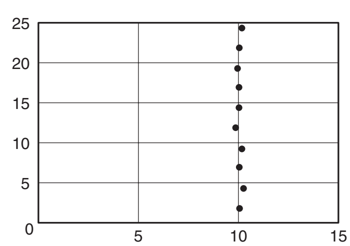
The plane is on the ground, at a height of 0 feet, at the beginning of the flight. The plane takes off and gains altitude quickly. It then flies at the same altitude for a while. The plane climbs to a higher altitude and flies at that height for a shorter time. The plane quickly descends and then lands.

What Is a Correlation in a Scatterplot?

One way to represent a set of related data is to graph the data using a scatterplot. In a **scatterplot** each pair of corresponding values in the data set is represented by a point on a graph.



To make predictions using a scatterplot, look for a **correlation,** or pattern, in the data. The pattern may not be true for every point, but look for the overall pattern the data seem to best fit.

As you move from left to right on the graph, if the data points …	as shown in this scatterplot …	they show this type of correlation:
move up		positive correlation
move down		negative correlation
show no pattern		no correlation
show a horizontal or vertical pattern	 	undefined correlation

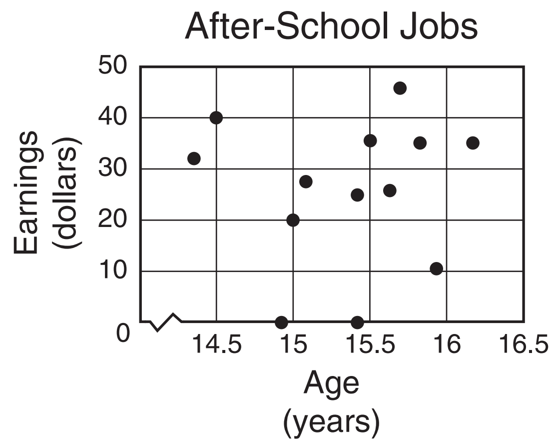
Ten employees at a shirt factory decorate shirts with hand-painted designs. The scatterplot below shows the relationship between the number of shirts each employee decorated last week and the number of hours each employee worked.

Describe the correlation between the number of shirts painted and the number of hours worked.

Look for a pattern in the graph. As you move from left to right, the data points move up. The general tendency is for the number of shirts to increase as the number of hours an employee works increases. The graph shows a positive correlation between the number of shirts painted and the number of hours worked.

Try It

A survey of ninth-grade students was conducted to see whether there was a correlation between their ages and the number of dollars they earned per week in after-school jobs. The data are presented in the scatterplot below.



Describe the correlation between students' ages and their weekly earnings.

As you move from left to ____________________ on the graph, the data points show _____ pattern.

There is _____ correlation between students' ages and their weekly earnings.

As you move from left to right on the graph, the data points show no pattern. There is no correlation between students' ages and their weekly earnings.

How Do You Use Symbols to Represent Unknown Quantities?

Represent unknown quantities with **variables**, or letters such as x or y. Use these variables in expressions, equations, or inequalities.

Gil runs a commercial window-washing business. He charges $35 per window for large office windows. His assistant Harold can wash 1 more large office window per hour than Gil can wash. Write an algebraic expression that represents the income they can earn in 6 hours of washing large office windows.

- Represent the number of windows per hour each can wash.

$$x = \text{Gil}$$
$$x + 1 = \text{Harold}$$

- Represent the number of windows each can wash in 6 hours.

$$6x = \text{Gil}$$
$$6(x + 1) = \text{Harold}$$

- Represent their earnings: number of windows • $35/window.

$$6x(35) = \text{Gil}$$
$$6(x + 1)(35) = \text{Harold}$$

- Represent the sum of their earnings.

$$\text{Gil's earnings} + \text{Harold's earnings}$$
$$6x(35) + 6(x + 1)(35)$$

The expression $6x(35) + 6(x + 1)(35)$ represents the income Gil and Harold can earn.

Objective 2

The diameter of the larger of two circles is 4 centimeters greater than the diameter of the smaller circle. The smaller circle has a radius of r centimeters.

Write an expression that represents the difference between the circumferences of the larger and smaller circles.

	Smaller Circle	Larger Circle
• Represent the diameter of each circle.	$2r$ The diameter is twice the radius.	$2r + 4$ The diameter is 4 cm greater than the smaller circle's.
• Represent the circumference of each circle using the formula in the Mathematics Chart.	$\pi(2r)$	$\pi(2r + 4)$

• Represent the difference between the circumferences of the two circles.

$$\pi(2r + 4) - \pi(2r) =$$
$$2\pi r + 4\pi - 2\pi r =$$
$$4\pi$$

The expression 4π represents the difference between the circumferences of the larger and smaller circles.

MATHEMATICS

How Do You Represent Patterns in Data Algebraically?

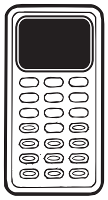
To represent patterns in data algebraically, follow these guidelines.

- Identify what quantities the data represent.
- Identify the relationships between those quantities.
- Look for patterns in the data.
- Use symbols to translate the patterns into an algebraic expression or equation.

A science class plants seedlings that are each 3 centimeters tall and monitors their growth over a four-week period. The average height of the seedlings at the end of each of the four weeks is given in the table below.

Seedling Heights

Week	1	2	3	4
Height (cm)	5	7	9	11



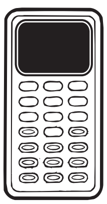
Write an expression that could represent the height of the seedlings after n weeks if the pattern continues.

The heights increase by a constant amount, 2 centimeters, each week. This is a linear relationship.

Look for a pattern. Each seedling starts at a height of 3 cm and grows 2 cm each week. The height of the seedlings in n weeks can be represented by the expression $2n + 3$. See whether this expression works for the known values.

Week (n)	$2n + 3$	Height (cm)	Yes/No
1	$2 \cdot 1 + 3 = 2 + 3 = 5$	5	Yes
2	$2 \cdot 2 + 3 = 4 + 3 = 7$	7	Yes
3	$2 \cdot 3 + 3 = 6 + 3 = 9$	9	Yes
4	$2 \cdot 4 + 3 = 8 + 3 = 11$	11	Yes

If the pattern continues, the height of the seedlings after n weeks could be represented by the expression $2n + 3$.



What rule expresses the functional relationship between x and y shown in the table below?

Change in x-values	x	y	Change in y-values
	6	33	
+3	9	78	+45
+3	12	141	+63
+3	15	222	+81

To find the rule, first see whether the relationship is linear. Compare the change in x-values to the change in y-values. For the same change in x-values, 3, there is a different change in y-values: 45, 63, 81. This function is not linear.

Next see whether the relationship is quadratic. Compare x^2 to y.

x	x^2	y
6	36	33
9	81	78
12	144	141
15	225	222

See whether all the coordinates satisfy the rule $y = x^2 - 3$. Substitute the values for x and y from the table into the equation.

x	$y = x^2 - 3$	y	Yes/No
6	$33 \stackrel{?}{=} 6^2 - 3$ $33 \stackrel{?}{=} 36 - 3$ $33 = 33$	33	Yes
9	$78 \stackrel{?}{=} 9^2 - 3$ $78 \stackrel{?}{=} 81 - 3$ $78 = 78$	78	Yes
12	$141 \stackrel{?}{=} 12^2 - 3$ $141 \stackrel{?}{=} 144 - 3$ $141 = 141$	141	Yes
15	$222 \stackrel{?}{=} 15^2 - 3$ $222 \stackrel{?}{=} 225 - 3$ $222 = 222$	222	Yes

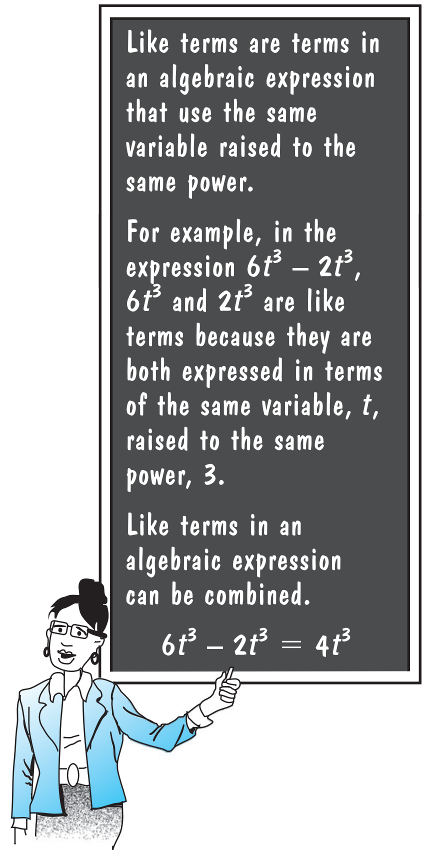
The ordered pairs satisfy the equation $y = x^2 - 3$.

How Do You Simplify Algebraic Expressions?

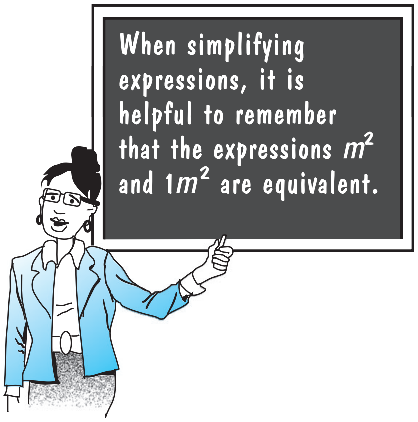
You can use the commutative, associative, and distributive properties to simplify algebraic expressions. Use these properties to remove parentheses and combine like terms.

Name	Property (If a, b, and c are any three real numbers, then…)	Examples
Commutative Property	$a + b = b + a$ or $ab = ba$	$3 + 4x = 4x + 3$ or $8r^2t = 8tr^2$
Associative Property	$a + (b + c) = (a + b) + c$ or $a(bc) = (ab)c$	$(2 + 6w) + 2w = 2 + (6w + 2w)$ $= 2 + 8w$ or $(3w)w = 3(w \cdot w)$ $= 3w^2$
Distributive Property	$a(b + c) = ab + ac$	$4(8x - 2) = 4(8x) + 4(-2)$ $= 32x - 8$

- Use the commutative property to change the order of the terms in addition and multiplication.
- Use the associative property to change the groupings in addition or multiplication.
- Use the distributive property to remove the parentheses by multiplying the term outside the parentheses by each term inside the parentheses.



Objective 2



Simplify the expression $m^2 + 5 - 5m^2 - 1$.

$$\begin{aligned} m^2 + 5 - 5m^2 - 1 &= (1m^2 - 5m^2) + (5 - 1) \\ &= -4m^2 + 4 \end{aligned}$$

When simplified, the expression $m^2 + 5 - 5m^2 - 1 = -4m^2 + 4$.

The height of a triangle is 2 inches more than 4 times its base. Write an expression in terms of b, the length of the base, that could be used to represent the area of the triangle. Write it in simplified form.

- The base of the triangle is b. The height of the triangle can be represented by the expression $4b + 2$.
- Substitute what you know into the formula for the area of a triangle.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}b(4b + 2)$$

$$A = \frac{1}{2}b(4b) + \frac{1}{2}b(2)$$

$$A = \frac{1}{2}(4)b^2 + \frac{1}{2}(2)b$$

$$A = 2b^2 + b$$

The area of the triangle can be represented by the expression $2b^2 + b$.

Do you see that . . .

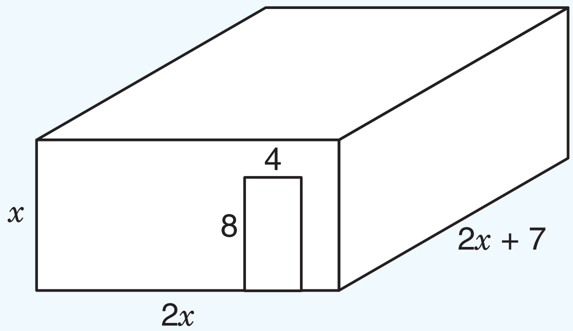
Simplify the expression $(5x^2 - 3x + 6) - (x^2 + 4x - 2)$.

When parentheses are preceded by a negative sign, it means the quantity in parentheses is multiplied by -1.

$$\begin{aligned} (5x^2 - 3x + 6) - (x^2 + 4x - 2) &= (5x^2 - 3x + 6) + -1(x^2 + 4x - 2) \\ &= 5x^2 - 3x + 6 - x^2 - 4x + 2 \\ &= (5x^2 - x^2) + (-3x - 4x) + (6 + 2) \\ &= 4x^2 - 7x + 8 \end{aligned}$$

When simplified, the expression equals $4x^2 - 7x + 8$.

On spring vacation the Hubble family helped build a medical clinic.



Write an expression that can be used to represent the number of square feet of plywood needed to cover the rectangular walls, not including the doorway.

- Represent each area using $A = bh$.

Area of Side Wall	Area of Front Wall	Area of Doorway
base $= 2x + 7$ height $= x$	base $= 2x$ height $= x$	base $= 4$ height $= 8$
$A = x(2x + 7)$ $A = 2x^2 + 7x$	$A = x(2x)$ $A = 2x^2$	$A = 4(8)$ $A = 32$

- Represent the combined area.

$A =$ Area of 2 side walls $+$ Area of 2 front walls $-$ Area of 1 doorway

$A = 2(2x^2 + 7x) + 2(2x^2) - 32$

$A = 4x^2 + 14x + 4x^2 - 32$

$$A = 8x^2 + 14x - 32$$

The expression $8x^2 + 14x - 32$ represents the number of square feet of plywood needed to cover the walls of the clinic.

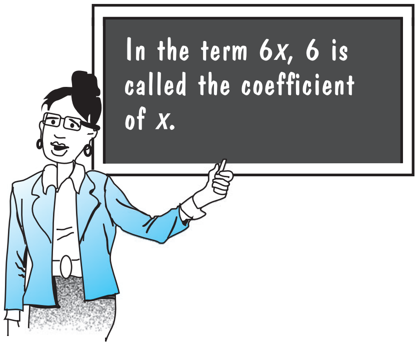
MATHEMATICS

Objective 2

How Do You Solve Algebraic Equations?

To solve algebraic equations, follow these guidelines.

- Simplify any expressions in the equation.
- Add or subtract on both sides of the equation to get variable terms on one side and constant terms on the other.
- Simplify again if necessary.
- Multiply or divide to obtain an equation that has the variable isolated with a coefficient of 1.



Solve the equation $2(3x + 1) = -10$.

$$\begin{aligned} 2(3x + 1) &= -10 \\ 2(3x) + 2(1) &= -10 \\ 6x + 2 &= -10 \\ -2 &= -2 \\ \hline 6x &= -12 \\ \frac{6x}{6} &= \frac{-12}{6} \\ x &= -2 \end{aligned}$$

In this equation, $x = -2$.

Solve the equation $4(b + 1) - 5 = 3b + 8$.

$$\begin{aligned} 4(b + 1) - 5 &= 3b + 8 \\ 4b + 4 - 5 &= 3b + 8 \\ 4b - 1 &= 3b + 8 \\ -3b \qquad &= -3b \\ \hline 1b - 1 &= 8 \\ +1 &= +1 \\ \hline b &= 9 \end{aligned}$$

In this equation, $b = 9$.

MATHEMATICS

Ron and Flo worked together making posters for the school election. Ron was able to make 4 posters per hour, while Flo made 6 posters per hour. Ron worked making posters for 1 hour longer than Flo.

Find the number of hours each person worked if together they made 29 posters.

- Let n represent the number of hours Flo worked.

  Let $n + 1$ represent the number of hours Ron worked.
- If Flo made 6 posters/hour, then she made $6n$ posters in n hours.

  If Ron made 4 posters/hour, then he made $4(n + 1)$ posters in $n + 1$ hours.
- Together they made $6n + 4(n + 1)$ posters.

  Together they made 29 posters.
- Write an equation and solve it.

$$\begin{aligned} 6n + 4(n + 1) &= 29 \\ 6n + 4n + 4 &= 29 \\ 10n + 4 &= 29 \\ 10n &= 25 \\ n &= 2.5 \end{aligned}$$

- Use this value to find the number of hours they each worked.

  n = the number of hours Flo worked, 2.5

  $n + 1$ = the number of hours Ron worked, $2.5 + 1$, or 3.5
- Check to see if the answer is correct.

  In 2.5 hours, Flo made $6(2.5) = 15$ posters.

  In 3.5 hours, Ron made $4(3.5) = 14$ posters.

  Since $15 + 14 = 29$, the answer is correct.

Flo worked for 2.5 hours, and Ron worked for 3.5 hours.

Gina planted some flowers she bought at a nursery. She planted 20 flowers each hour, working from 7 A.M. to 4 P.M. Gina took a lunch break at noon. If Gina planted 160 flowers, how long was her lunch break?

- Let b represent the number of hours in Gina's lunch break.
- Represent the number of hours Gina planted flowers.

  From 7 A.M. to 4 P.M. is 9 hours.

  Subtract b from 9 to find the number of hours Gina planted flowers.

  The expression $(9 - b)$ represents the number of hours Gina planted flowers.
- Write an equation.

  Gina planted 20 flowers per hour.

  The total number of flowers Gina planted, 160, is equal to the rate at which she planted flowers, 20 per hour, times the number of hours that she worked, $(9 - b)$.

  $160 = 20(9 - b)$
- Here are two different ways to solve the equation.

$$\begin{aligned}
160 &= 20(9 - b)\\
160 &= 180 - 20b\\
+20b &= \quad\; +20b\\
\hline
20b + 160 &= 180\\
-160 &= -160\\
\hline
20b &= 20\\
\frac{20b}{20} &= \frac{20}{20}\\
b &= 1
\end{aligned}$$

$$\begin{aligned}
160 &= 20(9 - b)\\
160 &= 180 - 20b\\
-180 &= -180\\
\hline
-20 &= -20b\\
\frac{-20}{-20} &= \frac{-20b}{-20b}\\
1 &= b
\end{aligned}$$

Gina took a 1-hour lunch break.

MATHEMATICS

Try It

Jan predicted that the total number of hours, t, she spent on homework this school year could be given by the equation $t = 150n + 75$, where n represents the average number of hours spent on homework each school night.

If Jan actually spent 375 hours on homework this school year, how many hours, on average, did she spend doing homework each school night?

Substitute ______ for t in the equation and solve for n.

$$375 = 150n + 75$$

$$\underline{-\square \quad = \quad\quad -\square}$$

$$\_\_\_\_\_\_ = 150n$$

$$\frac{300}{\square} = \frac{150n}{\square}$$

$$\_\_\_\_\_\_ = n$$

On average, Jan spent ______ hours per night on homework.

Substitute 375 for t in the equation and solve for n.

$$375 = 150n + 75$$

$$\underline{-\ 75 = \quad\quad -\ 75}$$

$$300 = 150n$$

$$\frac{300}{150} = \frac{150n}{150}$$

$$2 = n$$

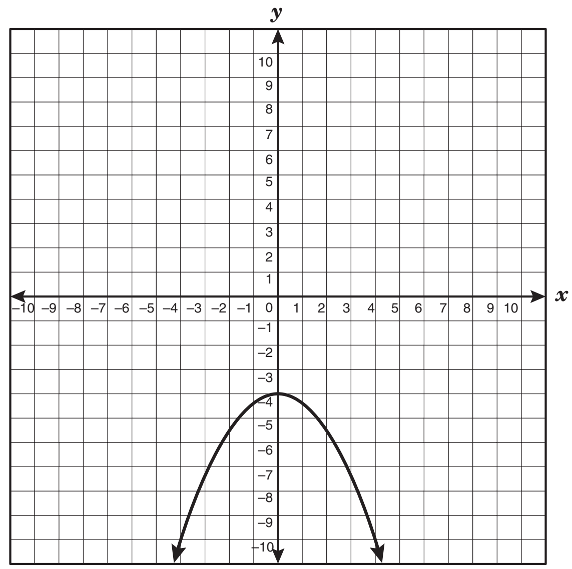
On average, Jan spent 2 hours per night on homework.

Now practice what you've learned.

Objective 2

Question 11

Which function is the parent function of the graph below?



A $y = -2x$

B $y = x - 4$

C $y = x^2$

D $y = x$

Answer Key: page 289

Question 12

A typical cashier at a grocery store can check customers out at a rate of 10 customers per hour. There are a total of 20 checkout stations; however, not all stations are necessarily operating at any one time. The total number of customers who can be checked out in an hour is given by the function $n = 10x$, where x is the number of cashiers working. What is a reasonable range for this function?

A $0 \leq n \leq 10$

B $0 \leq n \leq 20$

C $n \leq 200$

D $0 \leq n \leq 200$

Answer Key: page 289

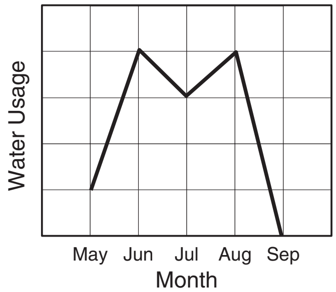
MATHEMATICS

Question 13

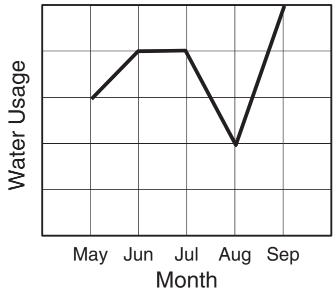
Jake keeps track of the amount of water he uses on his flower garden over the course of the summer. He finds that the less it rains, the more he needs to water the garden to keep his plants healthy and in bloom.

This summer the two driest months were June and August, but it rained so heavily in September that he didn't have to water his garden at all during that month. Which of the following graphs best represents Jake's water usage this summer?

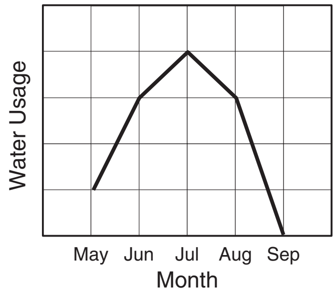
A



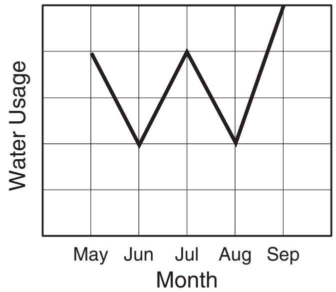
C



B



D

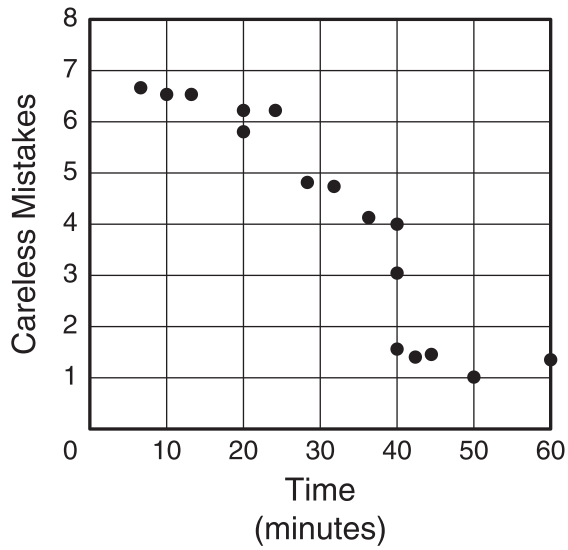


Answer Key: page 289

MATHEMATICS

Question 14

A math class was given 1 hour to complete a test. The scatterplot below shows the relationship between the speed at which the students completed the test and the number of careless mistakes they made.



Which of the following describes the correlation between the number of minutes spent on the test and the number of careless mistakes made?

A No correlation

B Positive correlation

C Negative correlation

D Undefined correlation

Answer Key: page 289

Question 15

Eliza is enrolled in a public-speaking class. Each week she is required to give a speech of greater length than the speech she gave the week before. The table below shows the lengths of several of her speeches.

Eliza's Speeches

Week Number	3	4	5	6
Length of Speech (seconds)	150	180	210	240

If this trend continues, in which week will she give a 12-minute speech?

A 22

B 12

C 15

D 24

Answer Key: page 289

Question 16

Brock is six feet tall. He climbs a ladder to paint some trim on his house. For each rung that he climbs, Brock is 1.2 feet higher above the ground. Which expression could you use to calculate the distance from the top of Brock's head to the ground if r represents the number of ladder rungs he has climbed?

A $1.2r + 6$

B $1.2r$

C $r + 6$

D $6r + 1.2$

Answer Key: page 290

Question 17

Which sequence uses the algebraic expression $4n + 5$ to describe the relationship between a term in the sequence and its position, n, in the sequence?

A 4, 9, 14, 19, 24, …

B 4, 8, 12, 16, 20, …

C 9, 13, 17, 21, 25, …

D 9, 10, 11, 12, 13, …

Answer Key: page 290

Question 18

Which algebraic expression best represents the relationship between a term in the sequence below and its position, n, in the sequence?

$$0, 3, 8, 15, 24, \ldots$$

A $n - 1$

B $n^2 - 1$

C $3n - 1$

D $2n^2 - 2$

Answer Key: page 290

Question 19

The length of a rectangle is 1 inch greater than its width. If its dimensions are doubled, its area increases by 36 square inches. Which equation could be used to find its dimensions?

A $4w^2 + 4w = 36$

B $3w + 3 = 36$

C $3w^2 + 3w = 36$

D $4w + 4 = 36$

Answer Key: page 290

Question 20

Which expression is equivalent to the following expression?

$$\frac{1}{2}x(4x - 6) + 3(x^2 - 1)$$

A $5x^2 - 3x + 3$

B $x^2 + 3x - 6$

C $5x^2 - 3x - 3$

D $-x^2 + 3x + 3$

Answer Key: page 290

Objective 3

The student will demonstrate an understanding of linear functions.

For this objective you should be able to

- represent linear functions in different ways and translate among their various representations; and
- interpret the meaning of slope and intercepts of a linear function and describe the effects of changes in slope and *y*-intercept in real-world and mathematical situations.

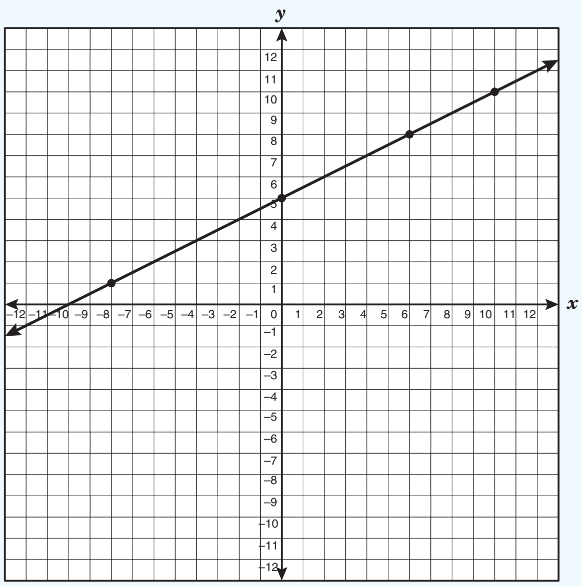
What Is a Linear Function?

A **linear function** is any function whose graph is a line.

Is the function represented by the following table of values a linear function?

x	y
−8	1
0	5
6	8
10	10

Graph the ordered pairs in the table on a coordinate grid.



The points lie on a line. Therefore, the function they represent is a linear function.

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Try It

The table below describes a linear relationship.

x	y
−6	8
−1	2
4	−4

Does the equation $6x + 5y = 4$ represent the same linear function?

To confirm that the equation $6x + 5y = 4$ represents the same linear function as the table, substitute the ordered pairs from the table into the equation.

x	y	Does $6x + 5y = 4$?	Yes/No
−6	8	$6(\_\_\_\_) + 5(\_\_\_\_) \stackrel{?}{=} 4$ $\_\_\_\_ + \_\_\_\_ \stackrel{?}{=} 4$ $\_\_\_\_ = 4$	____
−1	2	$6(\_\_\_\_) + 5(\_\_\_\_) \stackrel{?}{=} 4$ $\_\_\_\_ + \_\_\_\_ \stackrel{?}{=} 4$ $\_\_\_\_ = 4$	____
4	−4	$6(\_\_\_\_) + 5(\_\_\_\_) \stackrel{?}{=} 4$ $\_\_\_\_ + \_\_\_\_ \stackrel{?}{=} 4$ $\_\_\_\_ = 4$	____

The table and the equation represent the same function.

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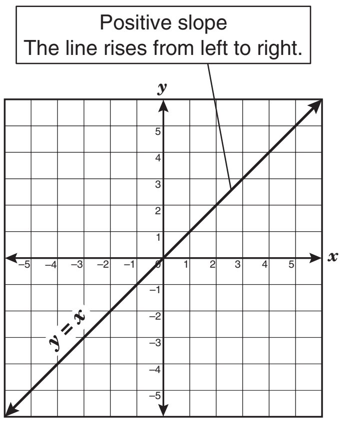
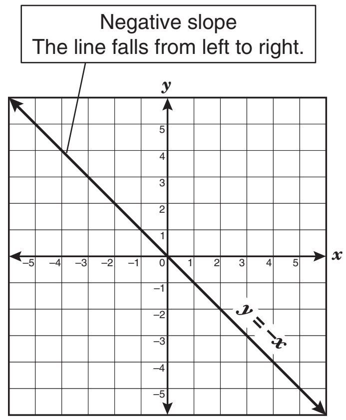
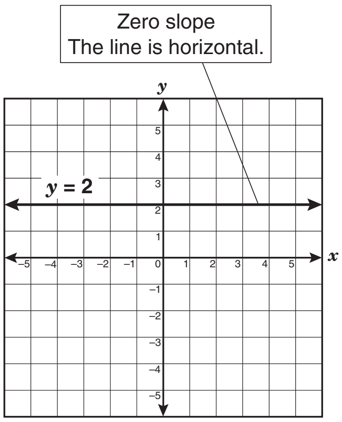
x	y	Does $6x + 5y = 4$?	Yes/No
−6	8	$6(-6) + 5(8) \stackrel{?}{=} 4$ $-36 + 40 \stackrel{?}{=} 4$ $4 = 4$	Yes
−1	2	$6(-1) + 5(2) \stackrel{?}{=} 4$ $-6 + 10 \stackrel{?}{=} 4$ $4 = 4$	Yes
4	−4	$6(4) + 5(-4) \stackrel{?}{=} 4$ $24 + -20 \stackrel{?}{=} 4$ $4 = 4$	Yes

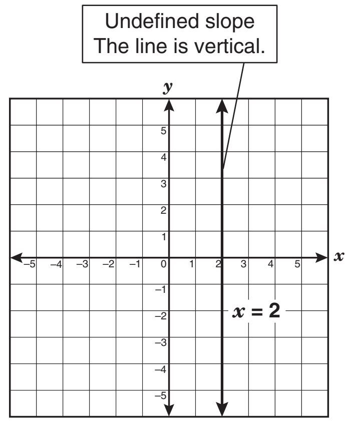
What Is Slope?

The **slope** of a graph is its rate of change, how fast it increases or decreases. The slope of a line can also be described as its steepness, how fast the line rises or falls.









The rate of change of a line is the ratio that compares the change in y-values to the corresponding change in x-values for any two points on the graph.

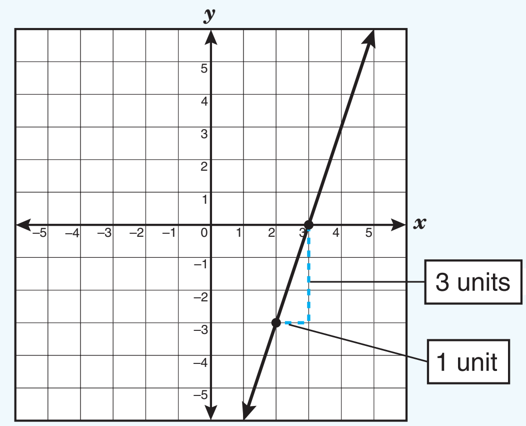
A graph's slope is often described as its **rise** (change in y) over **run** (change in x). To find the slope of a line from its graph:

- Pick any two points on the graph.
- Find the change in y-values, $y_2 - y_1$, or the rise.
  Count the number of units up or down between the two points.
- Find the change in x-values, $x_2 - x_1$, or the run.
  Count the number of units left to right between them.
- Determine whether the slope is positive or negative.
  As you go from left to right:
  if the line points up, the slope is positive.
  if the line points down, the slope is negative.
- Write the slope as a ratio: $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$.

Do you see that . . .

MATHEMATICS

In the graph below, you can find the slope by counting the change in y-values and the change in x-values between any two points.



The y-value changes 3 units for every 1 unit the x-value changes. As you go from left to right, the line points up, so the slope is positive. The rate of change is given by the ratio below.

$$\frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$$

The slope of the graph is 3.

Another way to find the slope is to use the slope formula.

Slope Formula

For any two points (x_1, y_1) and (x_2, y_2) on a graph, the slope, m, of the line that passes through them is:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \begin{array}{|l|} \hline \leftarrow \text{change in } y\text{-values} \\ \hline \leftarrow \text{change in } x\text{-values} \\ \hline \end{array}$$

What is the slope of the line passing through the points (2, 6) and (6, 3)?

Let (x_1, y_1) be (2, 6) and (x_2, y_2) be (6, 3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 6}{6 - 2} = \frac{-3}{4} = -\frac{3}{4}$$

The slope of the line is $-\frac{3}{4}$.

The set of ordered pairs below represents a linear function. What is the rate of change of the function?

$$\{(0, 0), (1, 7), (2, 14), (3, 21), (4, 28)\}$$

Since this is a linear function, you can use any two ordered pairs belonging to the function to find its rate of change.

- Use the ordered pairs (0, 0) and (2, 14).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 0}{2 - 0} = \frac{14}{2} = 7$$

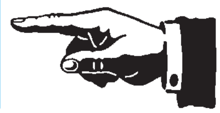
The rate of change of the function is 7.

- Use the ordered pairs (1, 7) and (4, 28).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{28 - 7}{4 - 1} = \frac{21}{3} = 7$$

The rate of change of the function is still 7.

The rate of change, or slope, of a linear function does not depend on which two points you pick to calculate the slope.



Do you see that . . .

Try It

What is the slope of the line passing through the points (0, 1) and (2, –1)?

Let (x_1, y_1) be (0, 1) and (x_2, y_2) be (2, –1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - \square}{2 - \square} = \frac{\square}{\square} = \_\_\_\_\_$$

If the slope is –1, then the graph's y-values decrease _____ unit for every 1 unit the x-values increase.

Let (x_1, y_1) be (0, 1) and (x_2, y_2) be (2, –1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{2 - 0} = \frac{-2}{2} = -1$$

If the slope is –1, then the graph's y-values decrease 1 unit for every 1 unit the x-values increase.

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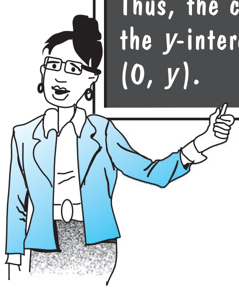
What Is the Slope-Intercept Form of a Linear Function?

One form of the equation of a linear function is $y = mx + b$. This form is called the **slope-intercept form** of a linear function.

When the equation of a linear function is written in the form $y = mx + b$:

- ***m*** is the **slope** of the graph of the function; m is the value by which the x-term is being multiplied.
- ***b*** is the ***y*-intercept** of the graph of the function; b is the value that is being added to the x-term.

The y-intercept of a graph is the y-coordinate of the point where the graph crosses the y-axis. The x-coordinate of any point on the y-axis is 0. Thus, the coordinates of the y-intercept are $(0, y)$.



What is the slope of the function $2y = 3x + 4$? What is the y-intercept of the function?

First write the equation in slope-intercept form, $y = mx + b$.

$$\begin{aligned} 2y &= 3x + 4 \\ \frac{2y}{2} &= \frac{3x}{2} + \frac{4}{2} \\ y &= \frac{3}{2}x + 2 \end{aligned}$$

Read the values of m and b from the equation.

For the function $y = \frac{3}{2}x + 2$:

- The value by which the x-term is being multiplied is $\frac{3}{2}$, so $m = \frac{3}{2}$.
- The value that is being added to the x-term is 2, so $b = 2$.

The slope of the function, m, is $\frac{3}{2}$.

The y-intercept of the function, b, is 2.

What are the slope and y-intercept of the function $4x + y = 5$?

- To determine the slope and y-intercept, transform the equation $4x + y = 5$ into slope-intercept form, $y = mx + b$.

$$\begin{array}{rcl} 4x + y &=& 5 \\ -4x &=& -4x \\ \hline y &=& -4x + 5 \end{array}$$

The equation is now in the form $y = mx + b$.

- Read the values of m and b from the revised equation. For this function, $m = -4$, and $b = 5$.

The slope of the function, m, is -4.

The y-intercept of the function, b, is 5.

Try It

Find the slope and y-intercept of the graph of the linear function $2x - 5y = 10$.

To determine the slope and y-intercept of the graph, transform the equation $2x - 5y = 10$ into slope-intercept form, $y = mx + b$.

$$\begin{array}{r} 2x - 5y = 10 \\ \underline{-2x \qquad = -2x} \end{array}$$

$$-5y = \_\_\_\_\_ + 10$$

$$\frac{-5y}{\square} = \frac{-2x}{\square} + \frac{10}{\square}$$

$$y = \frac{2}{\square}x - \_\_\_\_\_$$

Read the values of _____ and _____ from the revised equation.

For this function, $m =$ _____, and $b =$ _____. The slope of the function is _____. The y-intercept of the function is _____.

$$\begin{array}{r} 2x - 5y = 10 \\ \underline{-2x \qquad = -2x} \\ -5y = -2x + 10 \end{array}$$

$$\frac{-5y}{-5} = \frac{-2x}{-5} + \frac{10}{-5}$$

$$y = \frac{2}{5}x - 2$$

Read the values of m and b from the revised equation.

For this function, $m = \frac{2}{5}$, and $b = -2$.

The slope of the function is $\frac{2}{5}$. The y-intercept of the function is -2.

MATHEMATICS

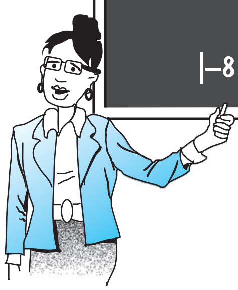
The absolute value of a number indicates its distance from 0 on a number line. The symbol for the absolute value of x is $|x|$.

For example,

$$|-3| = 3$$

$$|5| = 5$$

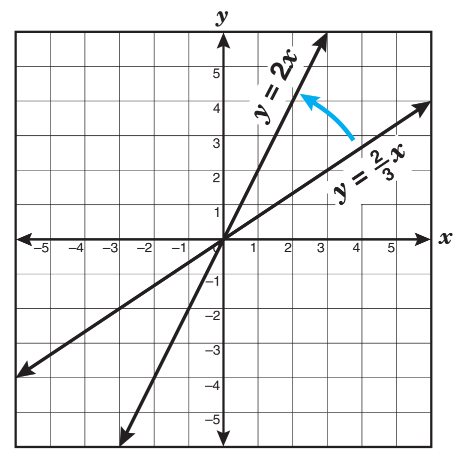
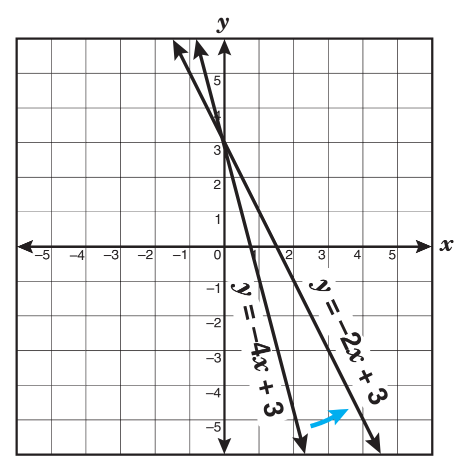
$$|-8.2| = 8.2$$



What Are the Effects on the Graph of a Linear Function If the Values of *m* and *b* Are Changed in the Equation *y* = *mx* + *b*?

In the equation $y = mx + b$, m represents the slope of the graph, and b represents the y-intercept of the graph. Changing either of these two constants, m or b, will affect the graph of the function.

Change in Slope, *m*

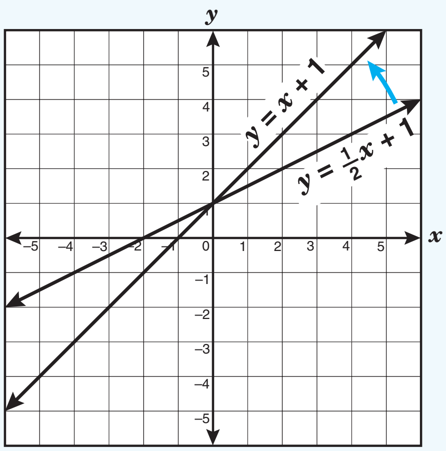
Function 1	Function 2	Effect
$y = \frac{2}{3}x$	$y = 2x$	 Since $\|2\| > \|\frac{2}{3}\|$, function 2 is steeper than function 1.
$y = -4x + 3$	$y = -2x + 3$	 Since $\|-2\| < \|-4\|$, function 2 is less steep than function 1.

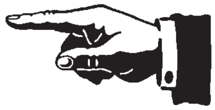
If the function $y = \frac{1}{2}x + 1$ were changed to $y = x + 1$, what would be the effect on the graph of the function?

The equations are both in slope-intercept form, $y = mx + b$. In this form, m represents the graph's slope.

- The slope of the original function is $\frac{1}{2}$.
- If the equation were changed to $y = x + 1$, the new value for m would be 1.

The graph of the new function would be steeper because $|1| > |\frac{1}{2}|$.

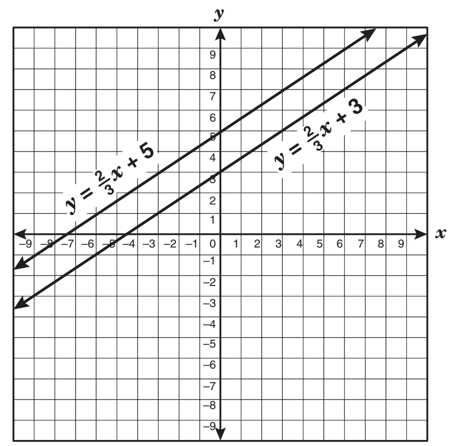
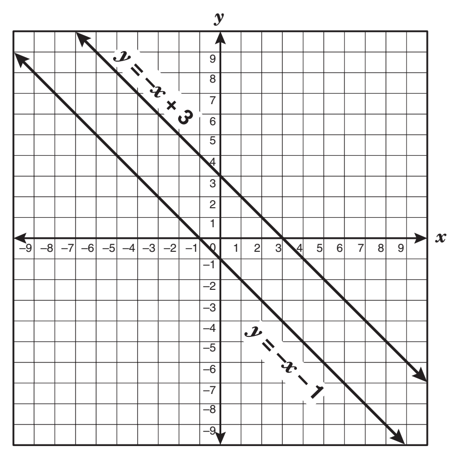




Do you see that . . .

MATHEMATICS

Change in *y*-intercept, *b*

Function 1	Function 2	Effect
$y = \frac{2}{3}x + 3$	$y = \frac{2}{3}x + 5$	 Function 2 crosses the *y*-axis at a higher point.
$y = -x + 3$	$y = -x - 1$	 Function 2 crosses the *y*-axis at a lower point.

If the *y*-intercept of the function $y = 3x - 6$ were increased by 10, what would be the equation of the new function?

The equation is in slope-intercept form, $y = mx + b$. In this form, b represents the graph's *y*-intercept.

- The *y*-intercept of the original function is -6.
- If the *y*-intercept of the function were increased by 10, it would be $-6 + 10 = 4$. The new value for b would be 4.

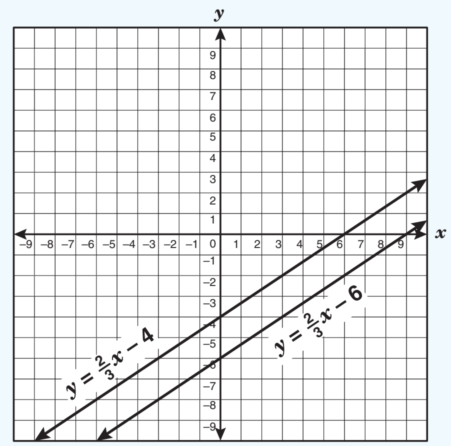
The equation of the new function would be $y = 3x + 4$.

The function $y = \frac{2}{3}x - 4$ was changed to $y = \frac{2}{3}x - 6$. What is the effect on the graph of the function?

When the equation of a linear function is written in the form $y = mx + b$, b represents the y-intercept of the graph.

- The value for b in the first function is -4.
  Its graph crosses the y-axis at $(0, -4)$.
- The value for b in the second function is -6.
  Its graph crosses the y-axis at $(0, -6)$.

Since $-6 < -4$, the graph of $y = \frac{2}{3}x - 6$ is 2 units below the graph of $y = \frac{2}{3}x - 4$.



Try It

If the y-intercept of the function $y = 2.5x + 7.25$ were increased by 1.75, what would be the effect on the graph of the function?

The equation of the given function is in slope-intercept form.

The slope of the line is ______, and its y-intercept is ______.

If the y-intercept were increased by 1.75, the new y-intercept would be equal to ______ + ______ = ______.

The new line would cross the y-axis at a ___________ point than the original line.

The new line would have the same slope, ______, as the original line.

The slope of the line is 2.5, and its y-intercept is 7.25. If the y-intercept were increased by 1.75, the new y-intercept would be equal to $7.25 + 1.75 = 9$. The new line would cross the y-axis at a higher point than the original line. The new line would have the same slope, 2.5, as the original line.

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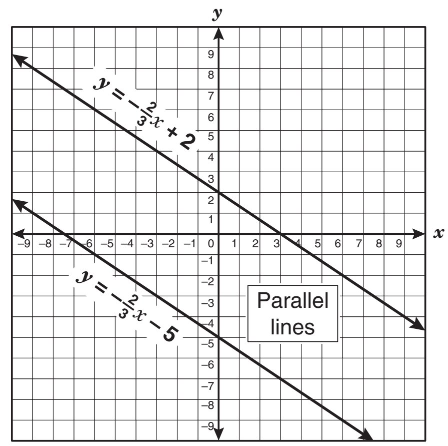
Slopes of lines can tell you whether the lines are parallel or perpendicular.

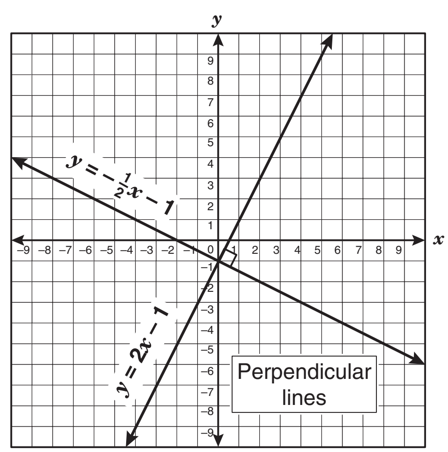
- If two lines are parallel, then they have the same slope and their equations have the same value for m.

  Look at the graphs of these two equations: $y = -\frac{2}{3}x - 5$ and $y = -\frac{2}{3}x + 2$.



- If two lines are perpendicular, then they have negative reciprocal slopes and their equations have negative reciprocal values for m.

  Look at the graphs of these two equations: $y = 2x - 1$ and $y = -\frac{1}{2}x - 1$.



Two numbers are negative reciprocals of each other if their product is -1. For example, $\frac{2}{3}$ and $-\frac{3}{2}$ are negative reciprocals.

- One fraction is the reciprocal of the other, and one fraction is the negative of the other.
- Since $\frac{2}{3} \cdot \frac{-3}{2} = \frac{-6}{6} = -1$, they are negative reciprocals.

MATHEMATICS

How Do You Write Linear Equations?

You can write linear equations in slope-intercept form, $y = mx + b$, or in **standard form**, $Ax + By = C$. In standard form, A, B, and C are integers, and A is usually greater than zero.

You can find the equation of a line given any of the following information:

- the slope and the y-intercept of the graph
- the slope and a point on the graph
- two points on the graph

<u>Given the slope and the y-intercept</u>

Identify the values for both m, the slope, and b, the y-intercept. Write the equation in slope-intercept form, $y = mx + b$, using these values.

What is the equation of the line with a slope of -2 and a y-intercept of 4?

Find the values you should substitute into the equation $y = mx + b$.

- If the slope is -2, then $m = -2$.
- If the y-intercept is 4, then $b = 4$.

The equation of the line is:

$$y = mx + b$$
$$y = -2x + 4$$

Try It

Write the equation of the line with a slope of $\frac{3}{5}$ and a y-intercept of -5.

If the slope is $\frac{3}{5}$, then $m = \frac{\square}{\square}$.

If the y-intercept is -5, then $b =$ ______.

The equation of the function is:

$$y = mx + b$$
$$y = \frac{\square}{\square}x + \_\_\_\_\_ \quad \text{or} \quad y = \frac{\square}{\square}x - \_\_\_\_\_$$

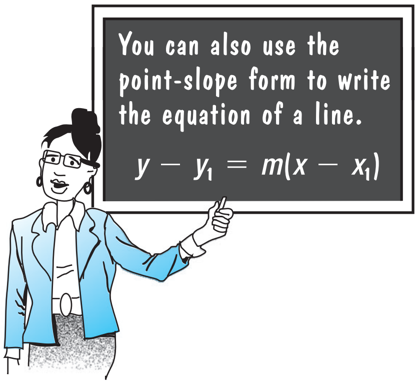
If the slope is $\frac{3}{5}$, then $m = \frac{3}{5}$. If the y-intercept is -5, then $b = -5$.

The equation of the function is: $y = \frac{3}{5}x + (-5)$ or $y = \frac{3}{5}x - 5$

Objective 3

Given the slope and a point on the graph

- Substitute the given values (the x- and y-coordinates of the given point and m, the slope) into the slope-intercept form of the equation, $y = mx + b$.
- Solve the equation for b.
- Substitute the values for m and b into the slope-intercept form, $y = mx + b$.



What is the equation of a line with a slope of 4, passing through the point $(3, -3)$?

- Substitute $x = 3$, $y = -3$, and $m = 4$ into the equation $y = mx + b$.

$$y = mx + b$$
$$-3 = 4(3) + b$$

- Solve for b.

$$\begin{aligned} -3 &= 4(3) + b \\ -3 &= 12 + b \\ -12 &= -12 \\ \hline -15 &= b \end{aligned}$$

- Substitute the given value for m, 4, and the value you have just found for b, -15, into the slope-intercept form of the equation.

$$y = mx + b$$
$$y = 4x - 15$$

The equation of the line is $y = 4x - 15$.

Do you see that . . .

This equation could also be written in standard form, $Ax + By = C$, in which A is usually positive.

$$\begin{aligned} y &= 4x - 15 \\ -4x &= -4x \\ \hline -4x + y &= -15 \end{aligned}$$

To change -4 to a positive value, multiply both sides of the equation by -1.

$$4x - y = 15$$

In standard form, the equation of this line is $4x - y = 15$.

Try It

Write the equation of the line that has a slope of $-\frac{1}{3}$ and contains the point (3, 2).

Begin by substituting the given values into the slope-intercept form.

The slope of the line is ______; therefore, $m =$ ______.

The line passes through the point (______, ______); therefore,

$x =$ ______ and $y =$ ______.

$$y = mx + b$$

$$\_\_\_\_\_\_ = \_\_\_\_\_\_(\_\_\_\_\_\_) + b$$

$$\_\_\_\_\_\_ = \_\_\_\_\_\_ + b$$

$$\_\_\_\_\_\_ = b$$

Finally, substitute the given value for m, ______, and the value you have just found for b, ______, into the slope-intercept form of the equation.

$$y = mx + b$$

$$y = \_\_\_\_\_\_x + \_\_\_\_\_\_$$

Begin by substituting the given values into the slope-intercept form. The slope of the line is $-\frac{1}{3}$; therefore, $m = -\frac{1}{3}$. The line passes through the point (3, 2); therefore, $x = 3$ and $y = 2$.

$$2 = -\frac{1}{3}(3) + b$$

$$2 = -1 + b$$

$$3 = b$$

Finally, substitute the given value for m, $-\frac{1}{3}$, and the value you have just found for b, 3, into the slope-intercept form of the equation.

$$y = -\frac{1}{3}x + 3$$

Given two points on the graph

- Use the x-coordinates and y-coordinates of the two given points to find the slope, m, of the line.
- Substitute the coordinates of one of the known points and the slope you just found into the slope-intercept form of the equation, $y = mx + b$.
- Solve the equation for b.
- Substitute m and b into the slope-intercept form, $y = mx + b$.

Find the equation of the line passing through the points (2, 6) and (7, 16).

- Begin by finding the slope of the graph using the slope formula. The line passes through the points (2, 6) and (7, 16).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16 - 6}{7 - 2} = \frac{10}{5} = 2$$

  If m is 2, the slope of the line is 2.

- Next substitute the coordinates of either of the given points and the value you just found for m, 2, into the slope-intercept form of the equation. Find the value of b.

  Use the coordinates (2, 6) or (7, 16).

$x = 2, y = 6$	$x = 7, y = 16$
$y = mx + b$	$y = mx + b$
$6 = 2(2) + b$	$16 = 2(7) + b$
$6 = 4 + b$	$16 = 14 + b$
$-4 = -4$	$-14 = -14$
$2 = b$	$2 = b$

- You now know the values of m and b; substitute them into the slope-intercept form of the equation.

$$y = mx + b$$
$$y = 2x + 2$$

The equation of the line passing through the points (2, 6) and (7, 16) is $y = 2x + 2$.

Try It

Write the equation of the linear function that contains the points (16, −10) and (−8, −25).

Begin by finding the ___________ of the graph.

The line passes through the points (_____, _____) and (_____, _____).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square} = \frac{\square}{\square}$$

The slope of the line is _____.

Next substitute the coordinates of one of the given points, (16, _____), and the slope you just found, _____, into the slope-intercept form of the equation of a line to find the value of b.

$$y = mx + b$$

$$\_\_\_\_\_ = \_\_\_\_\_(\_\_\_\_\_) + b$$

$$-10 = \_\_\_\_\_ + b$$

$$\_\_\_\_\_ = b$$

Finally, substitute the values of m and b into the slope-intercept form.

$$y = \_\_\_\_\_x + \_\_\_\_\_$$

or $$y = \_\_\_\_\_x - \_\_\_\_\_$$

Begin by finding the slope of the graph. The line passes through the points (16, −10) and (−8, −25).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-25 - (-10)}{-8 - 16} = \frac{-15}{-24} = \frac{5}{8}$$

The slope of the line is $\frac{5}{8}$.

Next substitute the coordinates of one of the given points, (16, −10), and the slope you just found, $\frac{5}{8}$, into the slope-intercept form of the equation of a line to find the value of b.

$$-10 = \frac{5}{8}(16) + b$$

$$-10 = 10 + b$$

$$-20 = b$$

Finally, substitute the values of m and b into the slope-intercept form.

$$y = \frac{5}{8}x + (-20)$$

or $$y = \frac{5}{8}x - 20$$

How Do You Find the *x*-intercept and *y*-intercept for an Equation?

Finding the x-intercept

The x-intercept is the point where the graph crosses the x-axis. The x-intercept has the coordinates $(x, 0)$.

To find the x-intercept, substitute $y = 0$ into the equation and solve for x. The value of x is the x-intercept. The graph of the line crosses the x-axis at the point $(x, 0)$.

What is the x-intercept of the graph of $x + 4y = 12$?

Substitute $y = 0$ into the equation and solve for x.

$$\begin{aligned} x + 4y &= 12 \\ x + 4(0) &= 12 \\ x + 0 &= 12 \\ x &= 12 \end{aligned}$$

The x-intercept of the graph is 12. The graph of the line crosses the x-axis at $(12, 0)$.

Finding the y-intercept

The y-intercept is the point where the graph of a line crosses the y-axis. The y-intercept has the coordinates $(0, y)$.

One way to find the y-intercept is to write the equation in slope-intercept form, $y = mx + b$. The value of b is the y-intercept. The graph of the line crosses the y-axis at the point $(0, b)$.

Another way to find the y-intercept is to substitute $x = 0$ into the equation and solve for y.

Here are two ways to find the y-intercept of the graph of $5y - 10 = 3x$.

Write the equation in slope-intercept form, $y = mx + b$.

$$\begin{aligned} 5y - 10 &= 3x \\ +10 &= \quad +10 \\ \hline 5y &= 3x + 10 \\ \frac{5y}{5} &= \frac{3x}{5} + \frac{10}{5} \\ y &= \frac{3}{5}x + 2 \end{aligned}$$

Therefore, $b = 2$.

Substitute $x = 0$ and solve for y.

$$\begin{aligned} 5y - 10 &= 3x \\ 5y - 10 &= 3(0) \\ 5y - 10 &= 0 \\ +10 &= +10 \\ \hline 5y &= 10 \\ \frac{5y}{5} &= \frac{10}{5} \\ y &= 2 \end{aligned}$$

The y-intercept of the graph is 2. The graph crosses the y-axis at $(0, 2)$.

What are the x- and y-intercepts of the line $10x + 5y = 30$?

To find the y-intercept, substitute $x = 0$ into the equation and solve for y.

$$\begin{aligned} 10x + 5y &= 30 \\ 10(0) + 5y &= 30 \\ 0 + 5y &= 30 \\ 5y &= 30 \\ \frac{5y}{5} &= \frac{30}{5} \\ y &= 6 \end{aligned}$$

The y-intercept is 6.

To find the x-intercept, substitute $y = 0$ into the equation and solve for x.

$$\begin{aligned} 10x + 5y &= 30 \\ 10x + 5(0) &= 30 \\ 10x + 0 &= 30 \\ 10x &= 30 \\ \frac{10x}{10} &= \frac{30}{10} \\ x &= 3 \end{aligned}$$

The x-intercept is 3.

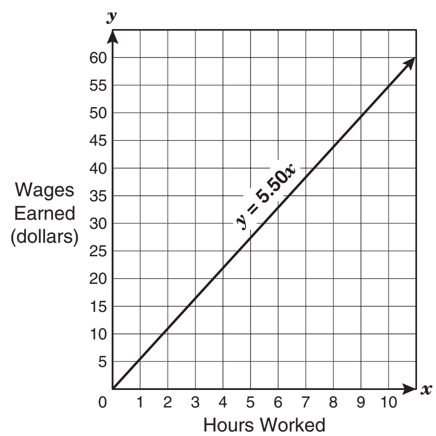
MATHEMATICS

How Do You Interpret the Meaning of Slopes and Intercepts?

To interpret the meaning of the slope or the *x*- or *y*-intercept of a function in a real-life problem, follow these guidelines.

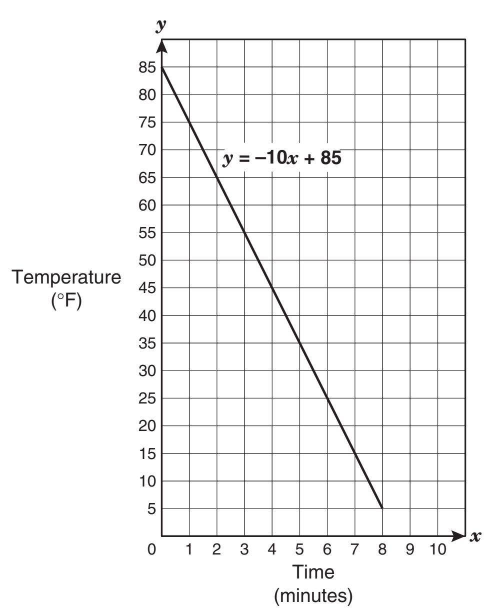
- The slope of the function is the function's rate of change. A graph's slope tells you how fast the function's dependent variable is changing for every unit change in the independent variable.

  For example, if a graph compares wages earned in dollars to hours worked, the slope tells the rate at which you are paid, or how much you make per hour.



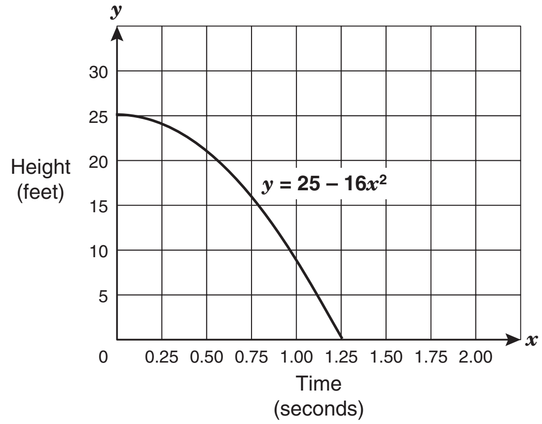
- The *y*-intercept is the point where the graph of a function crosses the *y*-axis. The *y*-intercept has the coordinates $(0, y)$. It is the point in the function where the independent quantity, x, has a value of 0. The *y*-intercept is often the starting point in a problem situation.

  For example, if a graph describes a constant temperature drop of 10°F per minute from $t = 0$ to $t = 8$ minutes, the *y*-intercept tells you what the temperature was at $t = 0$. The *y*-intercept of the graph is (0, 85), so the initial temperature was 85°F.



- The x-intercept is the point where the graph of the function crosses the x-axis. The x-intercept has the coordinates $(x, 0)$. It is the point in the problem where the dependent quantity, y, has a value of 0.

  For example, if a graph compares the height of a falling object to the number of seconds it has fallen, the x-intercept (when the object's height above the ground is 0) tells you how many seconds it will take for the object to hit the ground.



The growth in average U.S. income between 1976 and 2001 can be described by the following graph.

What do the slope and y-intercept of this graph tell you about the average U.S. income during this time period?

- The slope of the graph can be found by identifying the coordinates of two points on the graph and then using the slope formula to find the slope.
- The following two points are on the graph: (0, 16,000) and (25, 20,000).

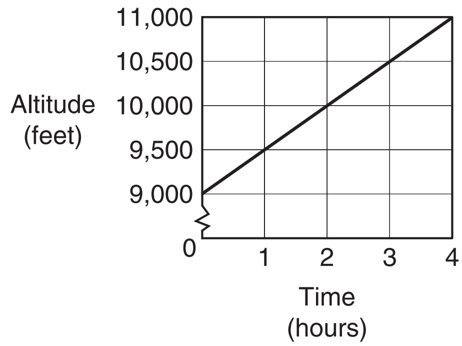
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20{,}000 - 16{,}000}{25 - 0} = \frac{4{,}000}{25} = 160$$

- The slope represents a change in average U.S. income of \$160 per year. Between the years 1976 and 2001, the average U.S. income increased at the rate of \$160 per year.
- The y-intercept of the line is at \$16,000. Since the x-axis represents the number of years since 1976, the y-intercept tells you that the average U.S. income in 1976 was \$16,000.

MATHEMATICS

Try It

Andy is climbing Long's Peak. The graph below shows his altitude in feet during the first four hours of the climb.



What does the slope of this graph tell you about Andy's climb?

Over the first four hours of his climb, Andy gained ______ feet of altitude.

The slope of the line is his ___________ in altitude divided by the ___________, or $\frac{\square \text{ feet}}{\square \text{ hours}}$.

The rate at which Andy gained altitude is ______ feet per hour.

The slope of the line tells you the ___________ at which Andy gained altitude.

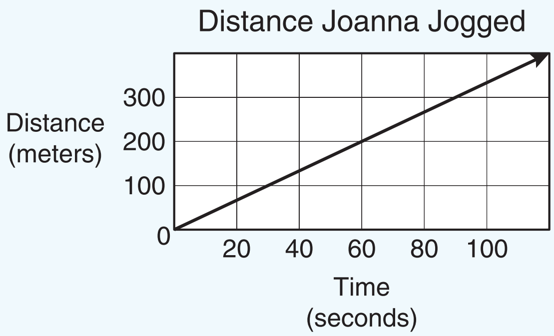
Over the first four hours of his climb, Andy gained 2,000 feet of altitude. The slope of the line is his increase in altitude divided by the time, or $\frac{2{,}000 \text{ feet}}{4 \text{ hours}}$. The rate at which Andy gained altitude is 500 feet per hour. The slope of the line tells you the rate at which Andy gained altitude.

How Do You Predict the Effects of Changing Slopes and *y*-intercepts in Applied Situations?

Many real-life problems can be modeled with linear functions. To analyze such problems, it is often helpful to identify the slope and the *y*-intercept of the linear function. Interpreting the meaning of these values will help you predict the effect that changing them will have on the quantities in the problem.

- If the slope is changed, a rate of change in the problem will increase or decrease.
- If the *y*-intercept is changed, an initial condition will change.

Joanna is on the track team. At practice she jogged steadily for 400 meters. If Joanna sprinted (ran faster) for the entire 400 meters instead of jogging, how would the graph change?



- Joanna jogged 400 meters in 120 seconds. So her speed was $\frac{400 \text{ meters}}{120 \text{ seconds}} = 3\frac{1}{3}$ meters per second.
- Joanna's speed is the slope of the line in the graph.
- When Joanna sprints, the rate at which she runs increases.
- Since the slope represents the speed at which Joanna ran, if she sprinted instead of jogged, the slope of the graph would increase.

The graph of the new line would be steeper.

Try It

Sarah volunteered to help a community organization wrap presents at the mall as a fund-raising event. When Sarah arrived to replace the person who was wrapping presents before her, she found that 135 presents had already been wrapped. Sarah was able to wrap 15 presents per hour over the three hours she volunteered.

How would a graph comparing the total number of boxes wrapped by the organization to the number of hours Sarah worked change if 150 presents had already been wrapped when she started?

Write an equation that represents the total number of presents wrapped. Let p represent the total number of presents wrapped, and let t represent the number of hours Sarah wrapped presents.

$$p = \_\_\_\_\_ t + \_\_\_\_\_$$

If 150 presents had already been wrapped when Sarah arrived, the equation would be $p = \_\_\_\_\_ t + \_\_\_\_\_$.

The graph of the second equation would cross the y-axis at a __________ point. The two graphs would have the same slope, _____, the rate at which Sarah wrapped packages. The graphs would be __________ lines.

Let p represent the total number of presents wrapped, and let t represent the number of hours Sarah wrapped presents: $p = 15t + 135$. If 150 presents had already been wrapped when Sarah arrived, the equation would be $p = 15t + 150$. The graph of the second equation would cross the y-axis at a higher point. The two graphs would have the same slope, 15, the rate at which Sarah wrapped packages. The graphs would be parallel lines.

How Do You Solve Problems Involving Direct Variation or Proportional Change?

If a quantity y varies directly with a quantity x, then the linear equation representing the relationship between the two quantities is $y = kx$. In this equation, k is called the **proportionality constant**.

To say "y varies directly with x" is to say "y is directly proportional to x."

If the equation $y = kx$ were graphed, k would be the slope of the graph.

Martin likes to cook for guests. He finds that the amount of time he spends cleaning the kitchen is directly proportional to the number of guests he serves. If it takes Martin 30 minutes to clean up the kitchen after serving 4 guests, how long will it take him to clean up after serving 10 guests?

- Write an equation that compares the number of minutes required to clean up to the number of guests.

  Let m = the number of minutes required to clean up.

  Let g = the number of guests.

  The direct variation equation is $m = kg$.

- Substitute the known values for m and g to find the proportionality constant, k. Use 30 minutes ($m = 30$) for 4 guests ($g = 4$).

$$m = kg$$

$$30 = k(4)$$

$$k = \frac{30}{4} = 7.5$$

  If $k = 7.5$, then the proportionality constant is 7.5.

  If $k = 7.5$, then the slope of the graph is 7.5.

- Find the number of minutes it will take to clean up after serving 10 guests by substituting $k = 7.5$ and $g = 10$ into the equation $m = kg$.

$$m = kg$$

$$m = 7.5(10)$$

$$m = 75$$

It would take Martin 75 minutes to clean up after serving 10 guests.

Now practice what you've learned.

MATHEMATICS

Question 21

To which of the following situations can the function $y = 5x + 10$ best be applied?

A The number of miles a person walks if he walks for 5 hours at the rate of 10 miles per hour

B The total weight on a scale if 5 pounds is placed there initially and a series of 10-pound weights are added to it

C The total wages earned by a waiter who is paid $5 per hour and earns $10 in tips

D The combined length of 5 boards, each 10 feet longer than the width of a doorway

Answer Key: page 291

Question 22

Which of the following equations describes the same function shown in the table below?

x	y
2	8
3	13
4	18
5	23

A $y = 5x - 2$

B $y = \frac{1}{5}x - 2$

C $y = 5x + 2$

D $y = \frac{1}{5}x + 2$

Answer Key: page 291

Question 23

What is the slope of the equation $2x - 5y = 10$?

A -2

B $\frac{2}{5}$

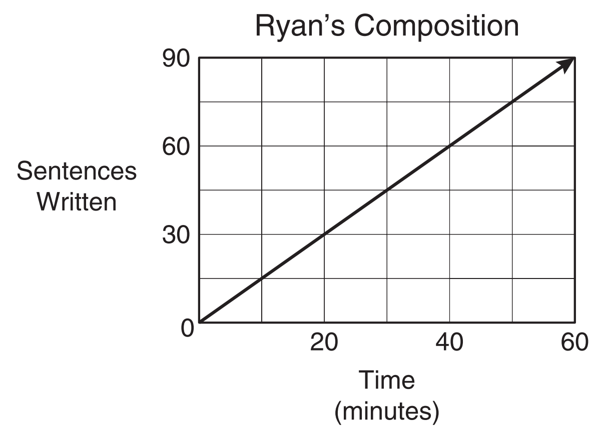
C 5

D $-\frac{2}{5}$

Answer Key: page 291

Question 24

Ryan is writing a composition for homework. He decides to keep track of the number of sentences he writes compared to the time in minutes he works. The graph below shows the data he collected.



At what rate does Ryan write his composition?

A 0.5 sentence per minute

B 1 sentence per minute

C 1.5 sentences per minute

D 2 sentences per minute

Answer Key: page 291

Question 25

What is the effect on the graph of the equation $6x + 3y = 12$ if 12 is changed to 36?

A The line is translated up 24 units.

B The line is translated up 8 units.

C The line is translated down 24 units.

D The line is translated down 8 units.

Answer Key: page 291

Question 26

Which statement best describes the relationship between the graphs of the equations $y = \frac{2}{3}x - 4$ and $3x + 2y = 12$?

A The graphs are two perpendicular lines.

B The graphs are two parallel lines.

C The graphs have the same y-intercept.

D The graphs have the same x-intercept.

Answer Key: page 291

Question 27

What is the equation of the line containing the points (7, 5) and (11, 9)?

A $y = 4x$

B $y = x - 2$

C $y = 2x - 2$

D $y = x + 2$

Answer Key: page 292

Question 28

What is the equation of a line with an x-intercept of -3 and a y-intercept of 5?

A $y = \frac{3}{5}x + 5$

B $y = \frac{5}{3}x + 5$

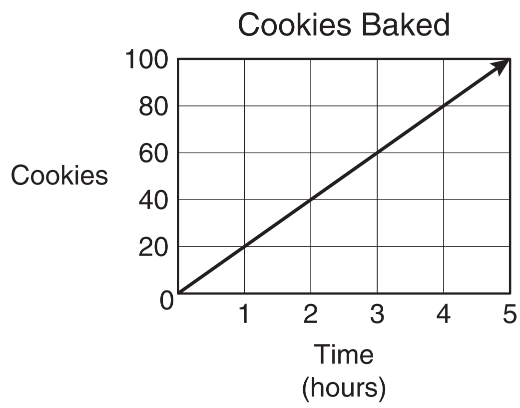
C $y = 5x + \frac{5}{3}$

D $y = -\frac{5}{3}x + 5$

Answer Key: page 292

Question 29

Mark and his friends are baking cookies for a bake sale. The graph below shows the total number of cookies they have compared to the number of hours they bake.



How would the graph change if Mark and his friends were given 20 cookies when they started baking?

A The y-intercept would increase.

B The slope would increase.

C The y-intercept would decrease.

D The slope would decrease.

Answer Key: page 292

MATHEMATICS

Question 30

A gardener knows that the number of potatoes harvested varies directly with the number of potato plants grown. Last year the gardener harvested 189 potatoes from 9 potato plants. If the gardener plants 14 potato plants this year, how many potatoes can he expect to harvest?

A 2646

B 23

C 294

D 21

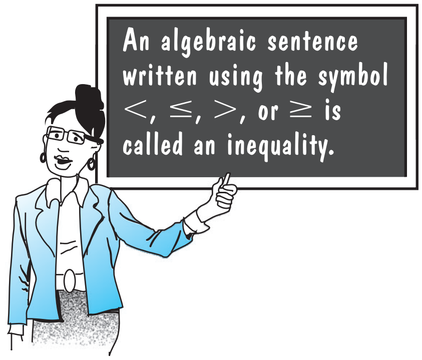
Answer Key: page 292

Objective 4

The student will formulate and use linear equations and inequalities.

For this objective you should be able to

- formulate linear equations and inequalities from problem situations, use a variety of methods to solve them, and analyze the solutions; and
- formulate systems of linear equations from problem situations.



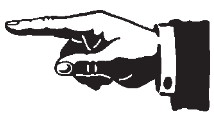
How Do You Solve Problems Using Linear Equations or Inequalities?

Many real-life problems can be solved using either a linear equation or an inequality. To solve the equation or inequality, follow these steps:

- Simplify the expressions in the equation or inequality by removing parentheses and combining like terms.
- Isolate the variable as a single term on one side of the equation by adding or subtracting expressions on both sides of the equation or inequality.
- Use multiplication or division to produce a coefficient of 1 for the variable term.
- When solving an inequality, you must reverse the inequality symbol if you multiply or divide both sides by a negative number.

$$-2x > 10$$
$$\frac{-2x}{-2} < \frac{10}{-2} \quad \text{Divide both sides by a negative number.}$$
$$x < -5$$

  The inequality symbol reversed; it went from $>$ to $<$.
- Use the solution of the equation or inequality to find the answer to the question asked.
- See whether your answer is reasonable.



Do you see that . . .

Chuck wants to put a rectangular vegetable garden in his yard and enclose it with a fence. The length of the garden is to be twice its width. Write an inequality Chuck could use to find the maximum width of the garden if he plans to use no more than 100 feet of fencing.

- Represent the quantities involved with variables or expressions. You know that the length of the garden is to be twice its width. Represent its width with w and its length with $2w$.

  The perimeter of the garden is the sum of its sides.

$$P = 2l + 2w$$

$$P = 2(2w) + 2w$$

- Write an inequality showing that the total number of feet of fencing is less than or equal to 100 feet.

$$\begin{aligned} P &\leq 100 \\ 2(2w) + 2w &\leq 100 \\ 4w + 2w &\leq 100 \\ 6w &\leq 100 \end{aligned}$$

The inequality $6w \leq 100$ could be used to find the maximum width of the garden.

Caroline is picking apples. She fills two baskets with apples and puts 7 apples in her pockets. The larger basket holds 25 more apples than the smaller one holds. If Caroline picks a total of 82 apples, how many apples does the smaller basket hold?

- Represent the quantities involved with variables or expressions.
  number of apples the smaller basket holds = n
  number of apples the larger basket holds = $n + 25$
  number of apples in her pockets = 7

- Write an equation that shows that Caroline picked a total of 82 apples.

$$n + (n + 25) + 7 = 82$$

- Solve the equation.

$$\begin{aligned} n + (n + 25) + 7 &= 82 \\ n + n + 25 + 7 &= 82 \\ 2n + 32 &= 82 \\ -32 &= -32 \\ \hline 2n &= 50 \\ n &= 25 \end{aligned}$$

In this problem, the solution to the equation, $n = 25$, is also the answer to the problem. The smaller basket holds 25 apples.

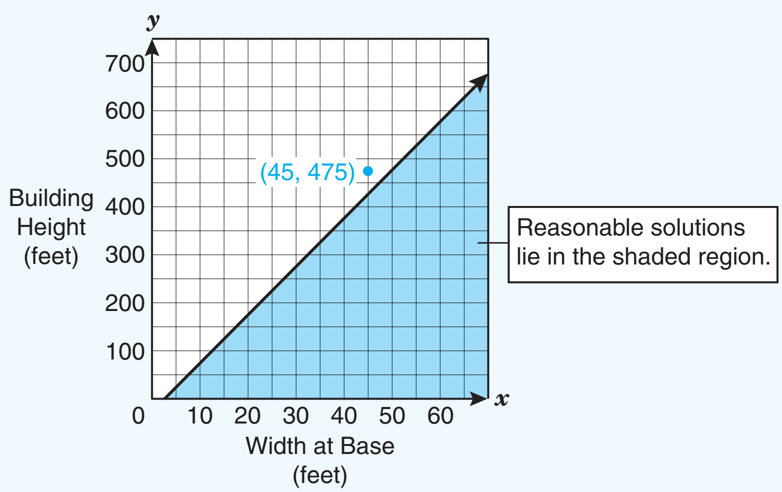
Tanya owns a beauty salon. Her weekly profits are given by the function $p = 40n - 630$, where n equals the number of customers she sees in 1 week. What is the minimum number of customers she must see in 1 week if she is to have a profit of at least $400?

Write an inequality that shows her profits greater than or equal to $400 then solve the inequality.

$$\begin{aligned} p &\geq 400 \\ 40n - 630 &\geq 400 \\ +630 &= +630 \\ \hline 40n &\geq 1030 \\ \frac{40n}{40} &\geq \frac{1030}{40} \\ n &\geq 25.75 \end{aligned}$$

Tanya must see at least 26 customers if she is to have a profit of at least $400 in 1 week.

An architect is designing a new office building. A 25-foot antenna will sit on top of the building. She wants the building's height, plus the antenna, to be no more than 10 times its width at its base. The possible height of the building, not including the antenna, is compared to its width at its base in the graph below.



Does a building with a width of 45 feet and a height of 475 feet, not including the antenna, meet the architect's requirements?

Find the value $x = 45$ on the x-axis and go up through the shaded region in the graph. Is the point (45, 475) in the shaded region? No, it is above the shaded region.

A building with a width of 45 feet and a height of 475 feet, not including the antenna, does not meet the architect's requirements. To meet the requirements, a building with a width of 45 feet would need to be 425 feet or shorter, not including the antenna.

Try It

In a triangle the longest side is 1 inch longer than twice the shortest side. The third side is 1 inch less than twice the shortest side. If the perimeter of the triangle is 40 inches, write an equation that could be used to find its dimensions.

Represent the lengths of the three sides of the triangle.

The shortest side can be represented by s.

The longest side is 1 inch longer than twice the shortest side,

_______s + _______.

The third side is 1 inch less than twice the shortest side,

_______s − _______.

Write an equation that shows the perimeter is __________ inches. Then simplify the equation.

Perimeter = $\text{length}_{\text{shortest side}} + \text{length}_{\text{longest side}} + \text{length}_{\text{third side}}$

_______ = s + (_______ + _______) + (_______ − _______)

_______ = ______________________

_______ = _______

The equation __________________ could be used to find the dimensions of the triangle.

The longest side is 1 inch longer than twice the shortest side, $2s + 1$. The third side is 1 inch less than twice the shortest side, $2s - 1$. Write an equation that shows the perimeter is 40 inches. Then simplify the equation.

$$40 = s + (2s + 1) + (2s - 1)$$
$$40 = s + 2s + 1 + 2s - 1$$
$$40 = 5s$$

The equation $40 = 5s$ could be used to find the dimensions of the triangle.

Try It

The Smiths were told that they would have to pay $56 for parts and an hourly labor charge to repair their washing machine. It took 2 hours to repair the washing machine. If the total cost for the repairs was $133, find the hourly rate the Smiths were charged for labor.

Represent the hourly labor rate with x.

The repairs took _______ hours.

The expression _______ represents the labor charge.

The parts cost $56.

The expression _______________ represents the total cost for the repairs.

Write an equation that shows that the total cost was $133 and solve it.

$$\begin{aligned} \_\_\_\_\_\_\_ x + \_\_\_\_\_\_\_ &= 133 \\ -56 &= -56 \\ \hline \_\_\_\_\_\_\_ x &= \_\_\_\_\_\_\_ \\ x &= \_\_\_\_\_\_\_ \end{aligned}$$

The Smiths were charged $_______ per hour for labor.

The repairs took 2 hours. The expression $2x$ represents the labor charge. The parts cost $56. The expression $2x + 56$ represents the total cost for the repairs. Write an equation that shows the total cost was $133 and solve it.

$$\begin{aligned} 2x + 56 &= 133 \\ -56 &= -56 \\ \hline 2x &= 77 \\ x &= 38.50 \end{aligned}$$

The Smiths were charged $38.50 per hour for labor.

MATHEMATICS

What Is a System of Linear Equations?

A **system of linear equations** is two or more linear equations that use two or more variables. Here is an example of a system of two linear equations.

$$x + 2y = 10$$
$$5x + y = 14$$

A **solution** to a system of linear equations is a pair of numbers for x and y that makes both equations true. The solution to the above system of linear equations is $x = 2$ and $y = 4$. These values for x and y make both equations true.

$$\begin{aligned} x + 2y &= 10 \\ \hline 2 + 2(4) &= 10 \\ 2 + 8 &= 10 \\ 10 &= 10 \end{aligned} \qquad \begin{aligned} 5x + y &= 14 \\ \hline 5(2) + 4 &= 14 \\ 10 + 4 &= 14 \\ 14 &= 14 \end{aligned}$$

Many real-life problems can be solved using a system of two or more linear equations.

How Do You Represent Problems Using a System of Linear Equations?

To represent a problem using a system of linear equations, follow these guidelines.

- Identify the quantities involved and the relationships between them.
- Represent the quantities involved with two different variables or with expressions involving two variables.
- Write two independent equations that can be used to solve the problem.

Laura has 29 salsa and country CDs. She has five more than three times as many salsa CDs as country CDs. Write a system of two linear equations with two unknowns that could be used to find the number of CDs of each type Laura has.

The problem involves two numbers. You could represent them using two different variables.

- Represent the number of salsa CDs with the variable s.
- Represent the number of country CDs with the variable c.

You know two different relationships between the numbers.

- Their sum is 29.

$$s + c = 29$$

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- The number of salsa CDs is five more than three times the number of country CDs.

salsa	is	five	more than	three	times	country
s	$=$	5	$+$	3	$\times$	c
s	$=$	5	$+$		$3c$	

The following system of linear equations can be used to find the two numbers.

$$s + c = 29$$
$$s = 5 + 3c$$

Try It

Diane is 8 years older than her sister. Phil is twice as old as his brother. Diane is 2 years younger than Phil. The sum of their four ages is 51. Write a system of linear equations that could be used to find their ages.

Represent their ages.

Diane's sister: s

Diane is 8 years older than her sister. Diane: __________.

Phil's brother: b

Phil is twice as old as his brother. Phil: __________.

Write two equations that describe the relationships between these quantities.

The sum of their four ages is 51.

$$s + (s + \underline{\qquad}) + b + (\underline{\qquad}b) = 51$$

Diane is 2 years younger than Phil.

$$s + \underline{\qquad} = 2b - \underline{\qquad}$$

Simplify each of the two equations.

$$\begin{aligned} s + (s + 8) + b + (2b) &= 51 \\ s + s + 8 + b + 2b &= 51 \\ \underline{\quad}s + \underline{\quad}b + 8 &= 51 \\ -\underline{\quad} &= -\underline{\quad} \\ \hline \underline{\quad}s + \underline{\quad}b &= \underline{\quad} \end{aligned}$$

$$\begin{aligned} s + 8 &= 2b - 2 \\ -8 &= -\underline{\quad} \\ \hline s &= 2b - \underline{\quad} \\ -2b &= -2b \\ \hline s - \underline{\quad}b &= -\underline{\quad} \end{aligned}$$

The following system of equations could be used to find their ages.

$$\underline{\qquad}s + \underline{\qquad}b = \underline{\qquad}$$
$$s - \underline{\qquad}b = \underline{\qquad}$$

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Diane: $s + 8$. Phil: $2b$.

The sum of their four ages is 51.

$$s + (s + 8) + b + (2b) = 51$$

Diane is 2 years younger than Phil.

$$s + 8 = 2b - 2$$

Simplify each of the two equations.

$$\begin{aligned} s + (s + 8) + b + (2b) &= 51 \\ s + s + 8 + b + 2b &= 51 \\ 2s + 3b + 8 &= 51 \\ -8 &= -8 \\ \hline 2s + 3b &= 43 \end{aligned}$$

$$\begin{aligned} s + 8 &= 2b - 2 \\ -8 &= \quad\; -8 \\ \hline s &= 2b - 10 \\ -2b &= -2b \\ \hline s - 2b &= -10 \end{aligned}$$

The following system of equations could be used to find their ages.

$$\begin{aligned} 2s + 3b &= 43 \\ s - 2b &= -10 \end{aligned}$$

Now practice what you've learned.

Question 31

Jemma and her cousin went to a restaurant for dinner. Jemma's dinner cost \$5 more than her cousin's. If their combined bill was under \$25, which inequality best describes the cost of their dinners?

A $x + 5 < 25x$

B $x + (x + 5) < 25$

C $x + (x + 25) < 5$

D $x - (x + 5) < 25$

Answer Key: page 293

Question 32

The population of Grandville is currently 15,400 people. If the population increases at an average rate of 325 people per year, which equation could be used to find the approximate number of years it will take for the population to reach 18,000 people?

A $15{,}400 + 325n = 18{,}000$

B $325n = 18{,}000$

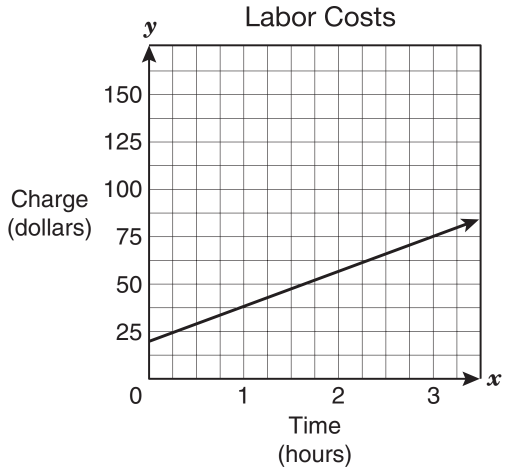
C $15{,}400n + 325 = 18{,}000$

D $15{,}400n = 18{,}000$

Answer Key: page 293

Question 33

Phil's Service Station uses the graph below to determine how much a mechanic should charge for labor for automobile repairs.



If the labor charge on an automobile repair bill was \$67.50, for approximately how many hours, h, did the mechanic work?

A $2.25 < h < 2.50$

B $2.75 < h < 3.00$

C $2.00 < h < 2.25$

D $2.50 < h < 2.75$

Answer Key: page 293

Question 34

The length of a rectangular area is 6 feet greater than the width. If the width is increased by 1 foot and the length remains the same, then the length of the new rectangle is twice the width. Find the dimensions of the original rectangle.

A 5 feet by 11 feet

B 5 feet by 10 feet

C 4 feet by 10 feet

D 4 feet by 8 feet

Answer Key: page 293

MATHEMATICS

Question 35

The gas tank in Karen's car holds 15 gallons. Her car gets between 25 and 30 miles to the gallon. If Karen fills up the gas tank and then drives until she runs out of gas, what is the least number of miles she can drive?

A 300 mi

B 375 mi

C 450 mi

D 405 mi

Answer Key: page 293

Question 36

At a linen sale Mrs. Green bought twice as many pillowcases for \$2 each as sheets for \$5 each. If she spent less than \$40, not including tax, what is the maximum number of pillowcases she could have purchased?

A 3

B 8

C 6

D 4

Answer Key: page 293

Question 37

Hector and Martha recently collected 32 new stamps. If Hector gives four of his new stamps to Martha, Martha will have three times as many new stamps as Hector. Which system of equations would allow you to calculate the number of new stamps Hector and Martha each have?

A $h + m = 32$
$3(h - 4) = m + 4$

B $h + m = 32$
$h - 4 = m + 4$

C $h = 32m$
$h - 4 = 3(m + 4)$

D $h - m = 32$
$3(h - 4) = m + 4$

Answer Key: page 294

Question 38

John lives 3.5 miles from school. One day he decided to run part of the way home and walk the rest of the way. If John walked six times as far as he ran, which system of equations can be used to find how many miles he ran?

A $w + r = 3.5$
$w - r = 6$

B $w + 6 = r$
$w + 3.5 = r$

C $w - r = 6$
$w = 6r$

D $w + r = 3.5$
$w = 6r$

Answer Key: page 294

MATHEMATICS

Objective 5

The student will demonstrate an understanding of quadratic and other nonlinear functions.

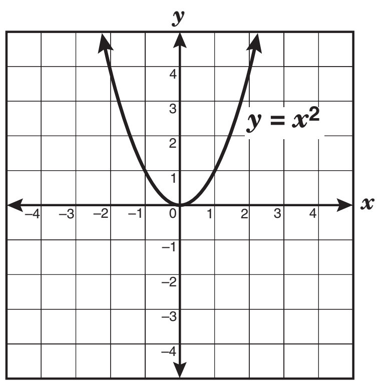
For this objective you should be able to

- interpret and describe the effects of changes in the parameters of quadratic functions; and
- apply the laws of exponents in problem-solving situations.

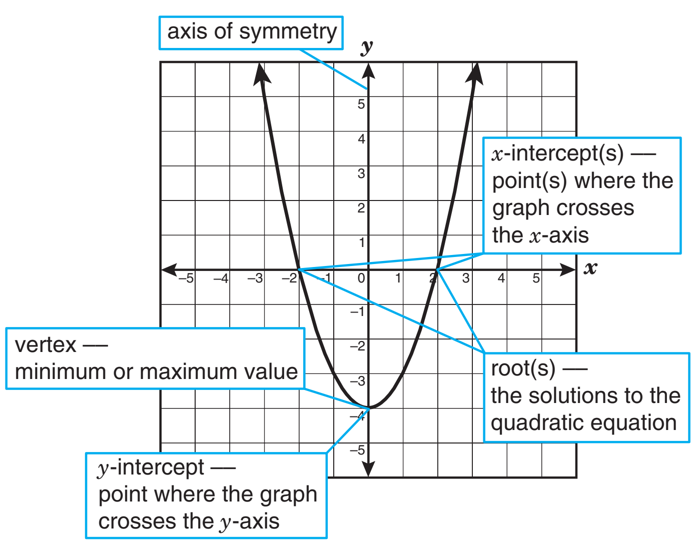
What Is a Quadratic Function?

- A **quadratic function** is any function whose graph is a parabola.
- A **quadratic equation** is any equation that can be written in the form $y = ax^2 + bx + c$. The constants a, b, and c are called the **parameters** of the equation. When you know their values, they help you describe the shape and location of the parabola.

The simplest quadratic function is $y = x^2$. It is the quadratic parent function.



The graph of the quadratic function $y = x^2 - 4$ is shown below.



MATHEMATICS

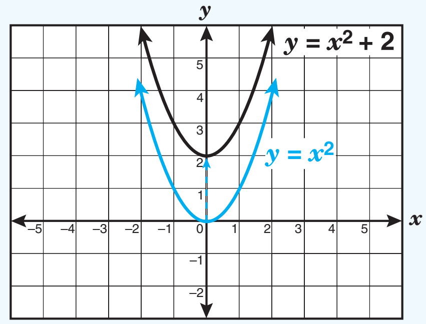
Objective 5

What Happens to the Graph of $y = x^2 + c$ When c Is Changed?

If two quadratic functions of the form $y = x^2 + c$ have different constants, c, then one graph will be a translation up or down of the other graph.

How does the graph of $y = x^2 + 2$ compare to the graph of $y = x^2$?

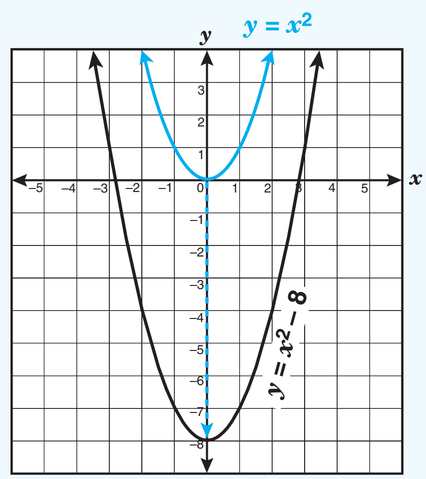
In the function $y = x^2 + 2$, the constant 2 has been added to the parent function $y = x^2$.



The graph of $y = x^2$ has been translated up 2 units.

How does the graph of $y = x^2 - 8$ compare to the graph of $y = x^2$?

In the function $y = x^2 - 8$, the constant 8 has been subtracted from the parent function $y = x^2$.

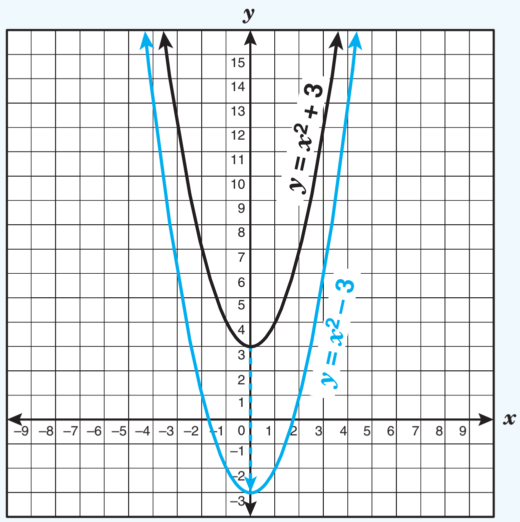


The graph of $y = x^2$ has been translated down 8 units.

MATHEMATICS

If the constant 3 in the function $y = x^2 + 3$ is changed to -3, how is the graph of the function affected?

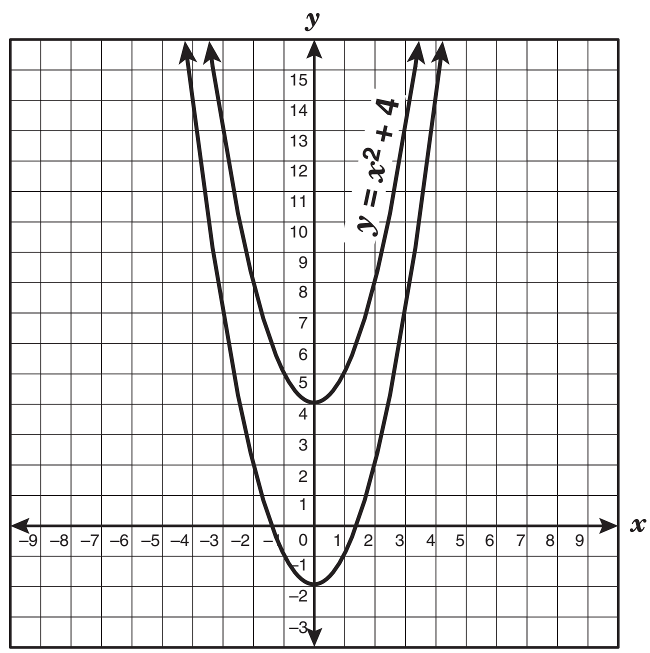
- Adding the constant 3 to the parent function $y = x^2$ translates the parent function's graph up 3 units.
- Changing the constant from 3 to -3 means that the new function is $y = x^2 - 3$. This translates the parent function's graph down 3 units.
- Look at the graphs of the two functions.



The vertex of the graph of $y = x^2 - 3$ is 6 units lower than the vertex of the graph of $y = x^2 + 3$.

Try It

The graph of the function $y = x^2 + 4$ is translated down 6 units. The equation $y = x^2 + 4$ and the translated function are graphed below.



What is the equation of the translated graph?

In the equation $y = x^2 + 4$, $c =$ __________.

If the graph is translated down 6 units, the value of c decreases __________ units.

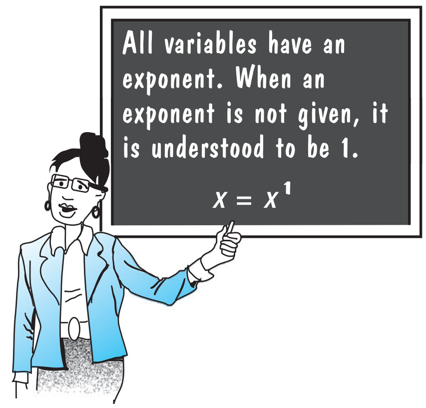
The value of c goes from __________ to __________.

The equation of the translated function is $y = x^2 -$ __________.

In the equation $y = x^2 + 4$, $c = 4$. If the graph is translated down 6 units, the value of c decreases 6 units. The value of c goes from 4 to -2. The equation of the translated function is $y = x^2 - 2$.

How Do You Apply the Laws of Exponents in Problem-Solving Situations?

When simplifying an expression with exponents, there are several rules, known as the **laws of exponents**, which must be followed.



- When multiplying terms with like bases, add the exponents.

$$x^a \cdot x^b = x^{(a+b)}$$

Example:

$$x^4 \cdot x^2 = x^{(4+2)} = x^6$$

$$(x \cdot x \cdot x \cdot x) \cdot (x \cdot x) = xxxxxx = x^6$$

- When dividing terms with like bases, subtract the exponents.

$$\frac{x^a}{x^b} = x^{(a-b)}$$

Example:

$$\frac{x^8}{x^3} = x^{(8-3)} = x^5$$

$$\frac{\cancel{x}\cancel{x}\cancel{x}xxxxx}{\cancel{x}\cancel{x}\cancel{x}} = x^5$$

Sometimes dividing variables with exponents produces negative exponents.

Example:

$$\frac{x^3}{x^5} = x^{(3-5)} = x^{-2}$$

$$\frac{\cancel{x}\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}\cancel{x}xx} = \frac{1}{x^2} = x^{-2}$$

- A term with a negative exponent is equal to the reciprocal of that term with a positive exponent.

$$x^{-a} = \frac{1}{x^a}$$

Example:

$$x^{-5} = \frac{1}{x^5}$$

- When raising a term with an exponent to a power, multiply the exponents.

$$(x^a)^b = x^{ab}$$

Example:

$$(x^2)^7 = x^{2 \cdot 7} = x^{14}$$

$$(x^2)^7 = (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx)$$

$$(x^2)^7 = (xxxxxxxxxxxxxx)$$

$$(x^2)^7 = x^{14}$$

- Any base other than zero raised to the zero power equals one.

$$x^0 = 1$$

Example:

$$8^0 = 1$$

Try It

Simplify the expression $5m^3 \cdot -2m$.

$$\begin{aligned} 5m^3 \cdot -2m &= 5 \cdot m^3 \cdot -2 \cdot m \\ &= (5 \cdot -2) \cdot (m^3 \cdot m) \\ &= -10 \cdot m^{(\square + \square)} \\ &= -10 \cdot m^{\square} \\ &= \underline{\qquad} \end{aligned}$$

$$\begin{aligned} 5m^3 \cdot -2m &= 5 \cdot m^3 \cdot -2 \cdot m \\ &= (5 \cdot -2) \cdot (m^3 \cdot m) \\ &= -10 \cdot m^{(3 + 1)} \\ &= -10 \cdot m^4 \\ &= -10m^4 \end{aligned}$$

Try It

Simplify the expression $(2x^4y^3)^5$.

$$\begin{aligned} (2x^4y^3)^5 &= 2^5 \cdot (x^4)^{\square} \cdot (y^3)^{\square} \\ &= 32 \cdot x^{(\square \cdot \square)} \cdot y^{(\square \cdot \square)} \\ &= 32 \cdot x^{\square} \cdot y^{\square} \\ &= 32x^{\square}y^{\square} \end{aligned}$$

$$\begin{aligned} (2x^4y^3)^5 &= 2^5 \cdot (x^4)^5 \cdot (y^3)^5 \\ &= 32 \cdot x^{(4 \cdot 5)} \cdot y^{(3 \cdot 5)} \\ &= 32 \cdot x^{20} \cdot y^{15} \\ &= 32x^{20}y^{15} \end{aligned}$$

The length of the base of a triangle is represented by the expression $2x^3y^2$, and its height is represented by the expression $7x^2y^4$. Represent its area with an expression in terms of x and y.

Substitute the expressions representing the lengths of the base and height into the formula for the area of a triangle and then simplify the expression using the properties of exponents.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(2x^3y^2)(7x^2y^4)$$
$$A = \frac{1}{2} \cdot 2 \cdot 7(x^3x^2)(y^2y^4)$$
$$A = 7x^{3+2}y^{2+4}$$
$$A = 7x^5y^6$$

Try It

Find the volume of a cube with a side length of $5m^2n$.

Substitute the expression representing the side length of a cube into the formula for the volume of a cube, $V = s^3$, and then simplify the expression.

$$V = s^3$$
$$V = (5m^2n)^{\square}$$
$$V = 5^{\square} \cdot (m^2)^{\square}(n)^{\square}$$
$$V = 125 \cdot m^{(2 \cdot \square)}n^{\square}$$
$$V = 125m^{\square}n^{\square}$$

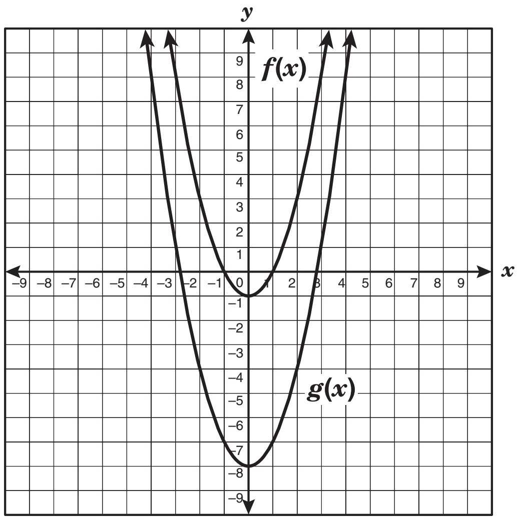
$$V = s^3$$
$$V = (5m^2n)^3$$
$$V = 5^3 \cdot (m^2)^3(n)^3$$
$$V = 125 \cdot m^{(2 \cdot 3)}n^3$$
$$V = 125m^6n^3$$

Now practice what you've learned.

MATHEMATICS

Question 39

The graphs of $f(x)$ and $g(x)$ are shown on the grid below.



If $f(x) = x^2 - 1$, what is the equation of $g(x)$?

A $g(x) = x^2 + 8$

B $g(x) = x^2 - 8$

C $g(x) = 8x^2 - 1$

D $g(x) = -8x^2 - 1$

Answer Key: page 294

Question 40

How does the graph of $f(x) = x^2 - 7$ compare to the graph of $g(x) = x^2 + 5$?

A The vertex of $f(x)$ is 12 units lower.

B The vertex of $f(x)$ is 12 units higher.

C The vertex of $f(x)$ is 2 units to the left.

D The vertex of $f(x)$ is 2 units to the right.

Answer Key: page 294

Question 41

The side length of a square is $4x^3yz^4$ units. What is the area of the square?

A $8x^6y^2z^8$ square units

B $8x^9yz^{16}$ square units

C $16x^6y^2z^8$ square units

D $16x^9yz^{16}$ square units

Answer Key: page 294

Question 42

The area of a parallelogram is $35p^6q^6$ square units. If the base of the parallelogram measures $5pq^2$ units, what is the height of the parallelogram? ($p \neq 0$ and $q \neq 0$)

A $7p^5q^4$ units

B $7p^6q^3$ units

C $30p^5q^4$ units

D $30p^6q^3$ units

Answer Key: page 294

Objective 6

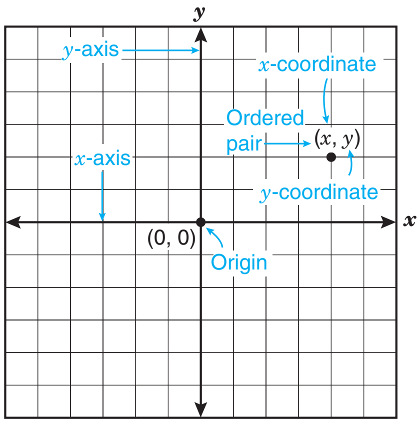
The student will demonstrate an understanding of geometric relationships and spatial reasoning.

For this objective you should be able to

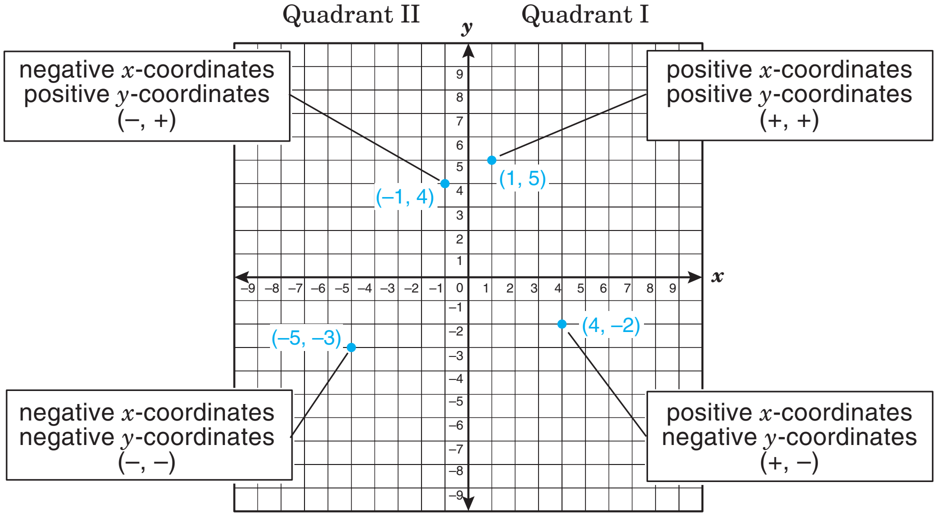
- use transformational geometry to develop spatial sense; and
- use geometry to model and describe the physical world.

How Do You Locate and Name Points on a Coordinate Plane?

A **coordinate grid** is used to locate and name points on a plane. A coordinate grid is formed by two perpendicular number lines.

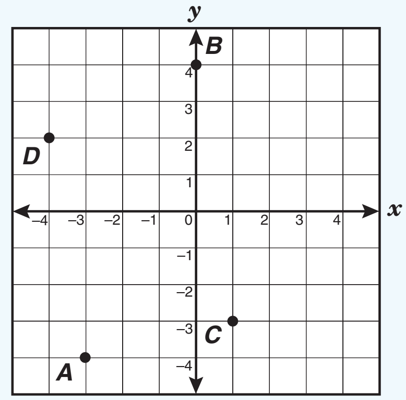


The x-axis and y-axis divide the coordinate plane into four regions called **quadrants**. The quadrants are usually referred to by the Roman Numerals I, II, III, and IV.



MATHEMATICS

Which of the points on the coordinate grid below satisfies the conditions $x < -2$ and $y > -3$?



Check to see whether the coordinates of the points satisfy the conditions.

- The y-coordinate must satisfy the inequality $y > -3$.
  Point A has the coordinates $(-3, -4)$. Its y-coordinate is -4.

  $$-4 < -3$$

  Point A is not correct.

- The x-coordinate must satisfy the inequality $x < -2$.
  Point B has the coordinates $(0, 4)$. Its x-coordinate is 0.

  $$0 > -2$$

  Point C has the coordinates $(1, -3)$. Its x-coordinate is 1.

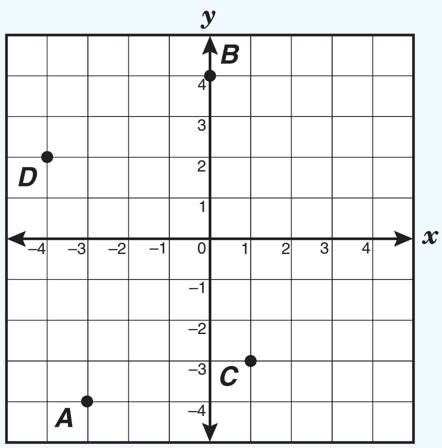
  $$1 > -2$$

  Points B and C are not correct.

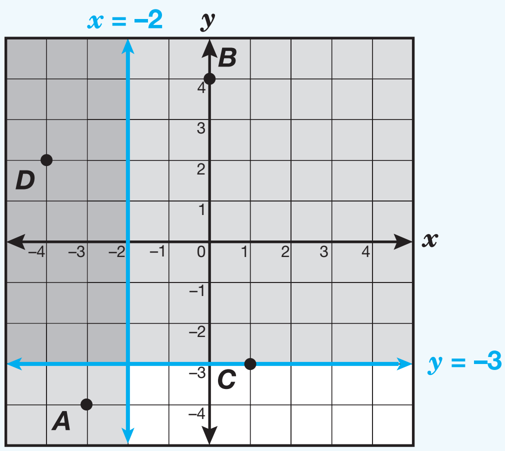
- Point D has the coordinates $(-4, 2)$.
  The x-coordinate is -4. Is $-4 < -2$? Yes. The first condition is satisfied.
  The y-coordinate is 2. Is $2 > -3$? Yes. The second condition is satisfied.

Only the coordinates of point D satisfy both conditions.

There is another way to find which of the points on the coordinate grid below satisfies the conditions $x < -2$ and $y > -3$.



- Draw a vertical line through $x = -2$. All the points to the left of this line have an x-coordinate less than -2.
- Draw a horizontal line through $y = -3$. All the points above this line have a y-coordinate greater than -3.

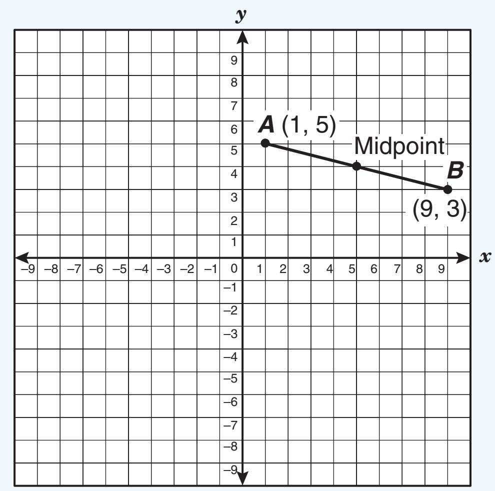


Only point D, with the coordinates $(-4, 2)$, satisfies both conditions.

How Do You Find the Midpoint of a Line Segment?

The **midpoint** of a line segment is the point that lies halfway between the segment's endpoints and divides it into two congruent parts. You can find the coordinates of the midpoint of a segment if you know the coordinates of its endpoints. Since the midpoint is halfway between the endpoints, its coordinates are the average of the coordinates of the endpoints.

Find the midpoint of $\overline{AB}$.



- To find the x-coordinate of the midpoint, find the x-value that is halfway between 1 and 9.

$$\frac{1 + 9}{2} = 5$$

The x-coordinate of the midpoint is 5.

- To find the y-coordinate of the midpoint, find the y-value that is halfway between 5 and 3.

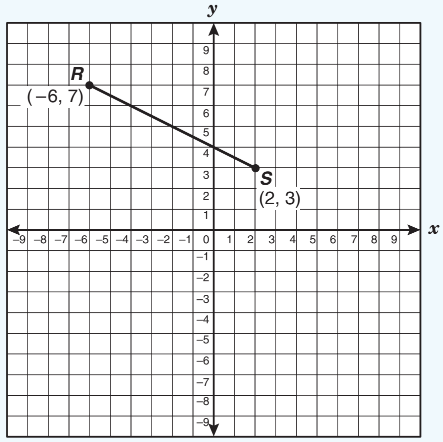
$$\frac{5 + 3}{2} = 4$$

The y-coordinate of the midpoint is 4.

The midpoint of $\overline{AB}$ is (5, 4).

MATHEMATICS

Find the midpoint of $\overline{RS}$.



- Look at the graph. Identify the coordinates of points *R* and *S*: *R* (−6, 7) and *S* (2, 3).
- To find the *x*-coordinate of the midpoint, find the *x*-value that is halfway between −6 and 2.

$$\frac{-6 + 2}{2} = \frac{-4}{2} = -2$$

The *x*-coordinate of the midpoint is −2.

- To find the *y*-coordinate of the midpoint, find the *y*-value that is halfway between 7 and 3.

$$\frac{7 + 3}{2} = \frac{10}{2} = 5$$

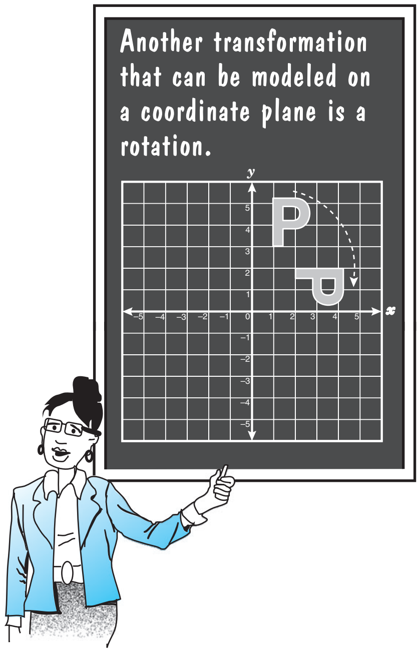
The *y*-coordinate of the midpoint is 5.

The midpoint of $\overline{RS}$ is (−2, 5).

MATHEMATICS

How Can You Show Transformations on a Coordinate Plane?

Translations, reflections, and dilations can all be modeled on a coordinate plane. A figure has been translated or reflected if it has been moved without changing its shape or size. A figure has been dilated if its size has been changed proportionally.



Translations

A **translation** of a figure is a movement of the figure along a line. It can be described by stating how many units to the left or right the figure is moved and how many units up or down it is moved. A figure and its translated image are always congruent.

The coordinates of point P are (m, n). What will be the point's new coordinates if it is translated 3 units to the right and 1 unit up?

- To translate a point 3 units to the right, add 3 to its x-coordinate.

  The x-coordinate of the point (m, n) is m.

  The x-coordinate of the translated point is $m + 3$.

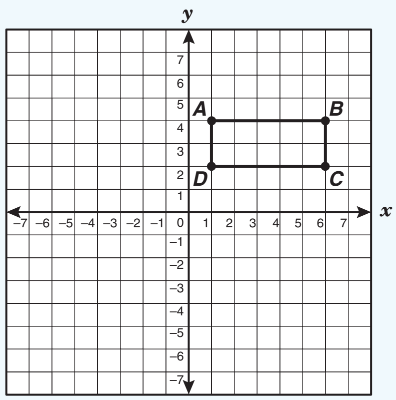
- To translate a point 1 unit up, add 1 to its y-coordinate.

  The y-coordinate of the point (m, n) is n.

  The y-coordinate of the translated point is $n + 1$.

The coordinates of the translated point are $(m + 3, n + 1)$.

If rectangle *ABCD* is translated 4 units to the left and 3 units down, what are the coordinates of the vertices of the translated rectangle $A'B'C'D'$?

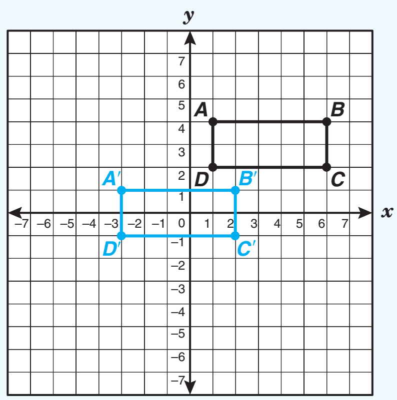


- The vertices of rectangle *ABCD* are *A* (1, 4), *B* (6, 4), *C* (6, 2), and *D* (1, 2).
- If the rectangle is translated 4 units to the left, then 4 must be subtracted from the *x*-coordinate of each vertex.

$$1 - 4 = -3$$
$$6 - 4 = 2$$
$$6 - 4 = 2$$
$$1 - 4 = -3$$

- If the rectangle is translated 3 units down, then 3 must be subtracted from the *y*-coordinate of each vertex.

$$4 - 3 = 1$$
$$4 - 3 = 1$$
$$2 - 3 = -1$$
$$2 - 3 = -1$$



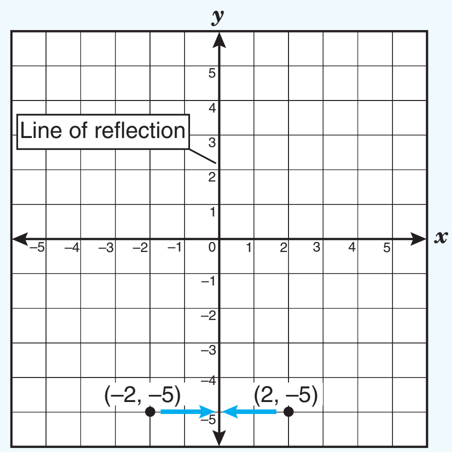
The vertices of the translated rectangle are A' (−3, 1), B' (2, 1), C' (2, −1), and D' (−3, −1).

Reflections

A **reflection** of a figure is the mirror image of the figure across a line. The line is called the **line of reflection**. The new figure is a reflection of the original figure, with the line of reflection serving as the mirror. A figure and its reflected image are always congruent.

Each point of the reflected image is the same distance from the line of reflection as the corresponding point of the original figure, but on the opposite side of the line of reflection.

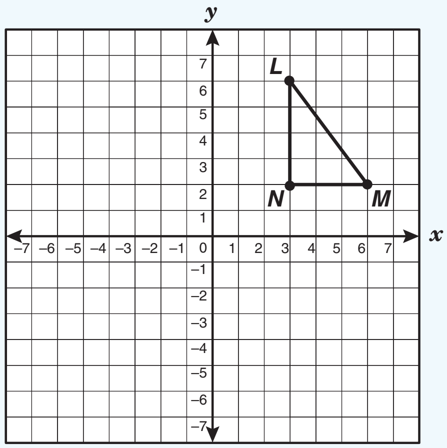
If the point $(2, -5)$ is reflected across the y-axis, what will be its new coordinates?



- The y-coordinate of the point will be unchanged, since the point is being reflected across the y-axis. The new point also will have a y-coordinate of -5.
- The x-coordinate of the point is 2 units to the right of the y-axis, so the x-coordinate of the new point will be 2 units to the left of the y-axis. The new point will have an x-coordinate of -2.

The coordinates of the reflected point will be $(-2, -5)$. The point $(2, -5)$ and its image $(-2, -5)$ are equally distant from the line of reflection, the y-axis.

If $\triangle LMN$ is reflected across the x-axis, what are the coordinates of its reflection, $\triangle L'M'N'$?



- The vertices of $\triangle LMN$ are L (3, 6), M (6, 2), and N (3, 2).
- Point L is 6 units above the x-axis, so L' is 6 units below the x-axis. The coordinates of L' are (3, −6).
- Point M is 2 units above the x-axis, so M' is 2 units below the x-axis. The coordinates of M' are (6, −2).
- Point N is 2 units above the x-axis, so N' is 2 units below the x-axis. The coordinates of N' are (3, −2).

The vertices of $\triangle L'M'N'$ are L' (3, −6), M' (6, −2), and N' (3, −2).

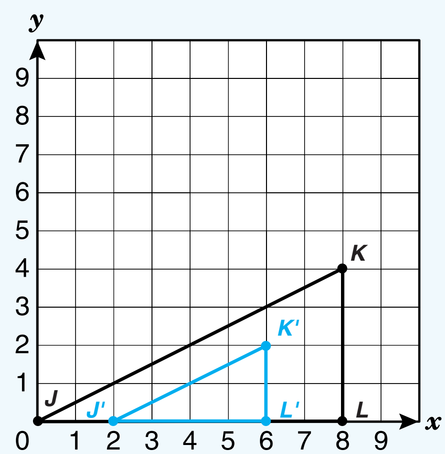
Dilations

A **dilation** is a proportional enlargement or reduction of a figure through a point called the center of dilation. The size of the enlargement or reduction is called the **scale factor** of the dilation.

- If the dilated image is larger than the original figure, then the scale factor > 1. This is called an **enlargement**.
- If the dilated image is smaller than the original figure, then the scale factor < 1. This is called a **reduction**.

A figure and its dilated image are always similar.

What scale factor was used to transform $\triangle JKL$ into $\triangle J'K'L'$?



To find the scale factor, compare the lengths of a pair of corresponding sides.

- Of the line segments that make up the triangles, it is easiest to find the lengths of $\overline{JL}$ and $\overline{J'L'}$ because they lie along the x-axis.
- The length of $\overline{JL}$ is the difference between the x-coordinates of points J and L.

$$JL = 8 - 0 = 8$$

- The length of $\overline{J'L'}$ is the difference between the x-coordinates of points J' and L'.

$$J'L' = 6 - 2 = 4$$

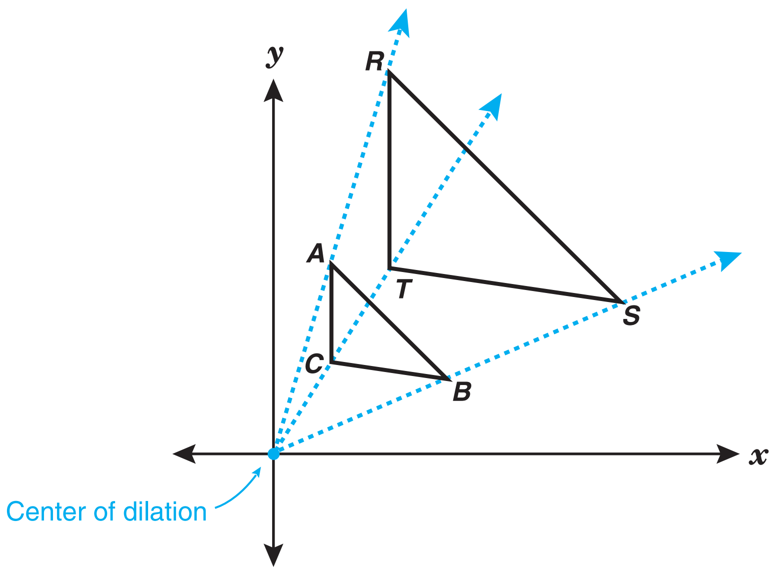
- The scale factor is the ratio of these lengths. Since the dilated image is smaller than the original image, the scale factor < 1.

$$\frac{J'L'}{JL} = \frac{4}{8} = \frac{1}{2}$$

The scale factor used to dilate $\triangle JKL$ to form $\triangle J'K'L'$ is $\frac{1}{2}$.

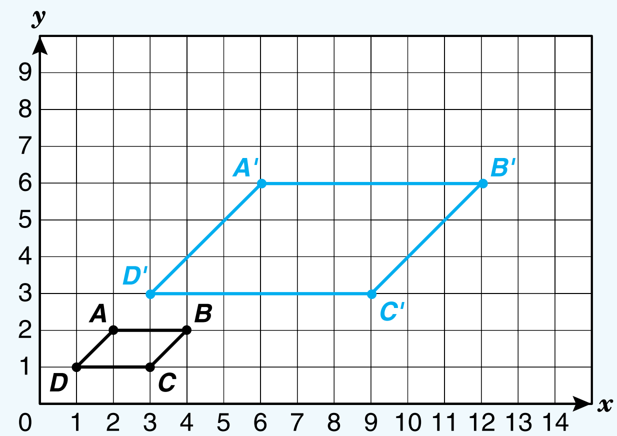
Each side of the dilated triangle is $\frac{1}{2}$ the length of the corresponding side of the original triangle.

Another way to view a dilation is as a projection through a center of dilation.



Looking at a dilation in this way can help you see it as an enlargement or a reduction. $\triangle ABC$ has been dilated to form $\triangle RST$.

Parallelogram $ABCD$ was dilated to form parallelogram $A'B'C'D'$ with (0, 0) as the center of dilation. What scale factor was used to dilate parallelogram $ABCD$?



To find the scale factor, compare the lengths of a pair of corresponding sides of the two parallelograms.

- Use the lengths of $\overline{AB}$ and $\overline{A'B'}$ to find the scale factor, since both segments are horizontal.
- The length of $\overline{AB}$ is the difference between the x-coordinates of points A and B.

$$AB = 4 - 2 = 2$$

- The length of $\overline{A'B'}$ is the difference between the x-coordinates of points A' and B'.

$$A'B' = 12 - 6 = 6$$

- From the graph you can tell that parallelogram $ABCD$ has been enlarged, so the scale factor > 1. Use this fact to be certain you state the ratio correctly.
- The scale factor is the ratio of the corresponding side lengths.

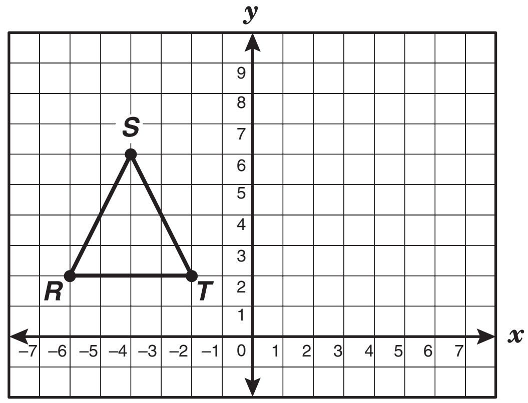
$$\frac{A'B'}{AB} = \frac{6}{2} = 3$$

A scale factor of 3 was used to dilate parallelogram $ABCD$.

MATHEMATICS

Try It

$\triangle RST$ has vertices R (−6, 2), S (−4, 6), and T (−2, 2). Find the coordinates of the vertices of its reflection across the y-axis.



Because this is a reflection across the y-axis, the

______-coordinates do not change.

The vertex R (−6, 2) is ______ units to the ____________ of the y-axis.

R' must be ______ units to the ____________ of the y-axis.

The coordinates of R' are (______, ______).

The vertex S (−4, 6) is ______ units to the ____________ of the y-axis.

S' must be ______ units to the ____________ of the y-axis.

The coordinates of S' are (______, ______).

The vertex T (−2, 2) is ______ units to the ____________ of the y-axis.

T' must be ______ units to the ____________ of the y-axis.

The coordinates of T' are (______, ______).

Because this is a reflection across the y-axis, the y-coordinates do not change. The vertex R (−6, 2) is 6 units to the left of the y-axis. R' must be 6 units to the right of the y-axis. The coordinates of R' are (6, 2). The vertex S (−4, 6) is 4 units to the left of the y-axis. S' must be 4 units to the right of the y-axis. The coordinates of S' are (4, 6). The vertex T (−2, 2) is 2 units to the left of the y-axis. T' must be 2 units to the right of the y-axis. The coordinates of T' are (2, 2).

Now practice what you've learned.

MATHEMATICS

Question 43

A quadrilateral has side lengths of 3, 3, 5, and 7 units. Which of the following could be the lengths of the sides of a dilation of this quadrilateral?

A 3, 5, 5, and 8 units

B 4, 4, 6, and 9 units

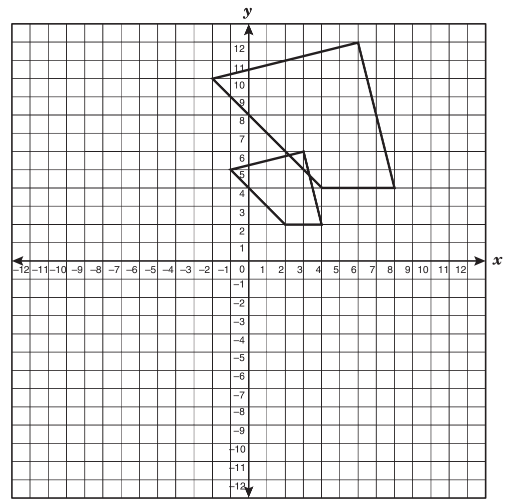
C 6, 6, 10, and 14 units

D 9, 9, 15, and 24 units

Answer Key: page 295

Question 44

Look at the figures shown below.



The figures best illustrate which of the following transformations?

A Rotation

B Dilation

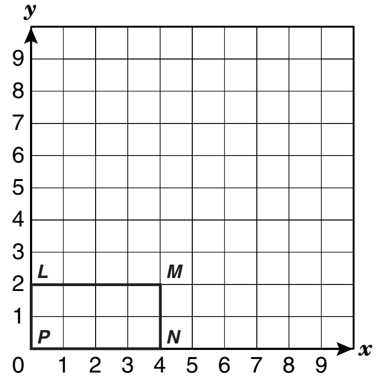
C Reflection

D Translation

Answer Key: page 295

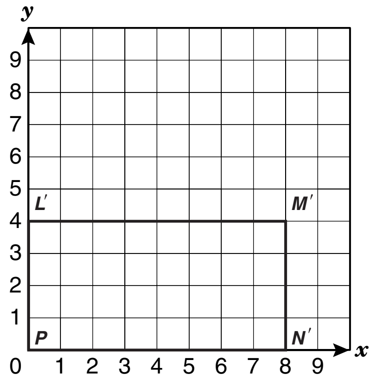
Question 45

The rectangle in the graph below will be dilated by a scale factor of 2.

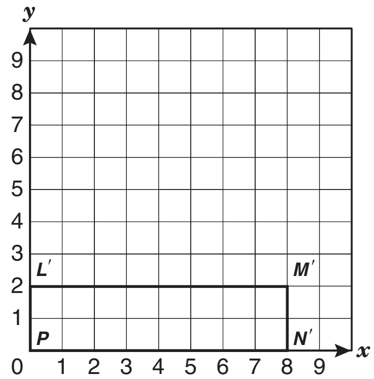


Which graph shows this dilation?

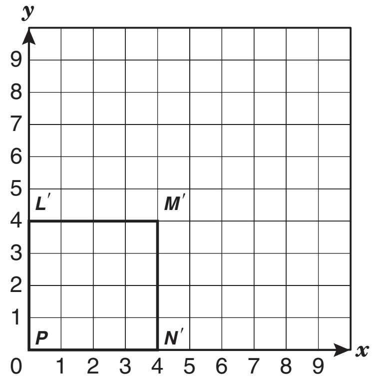
A



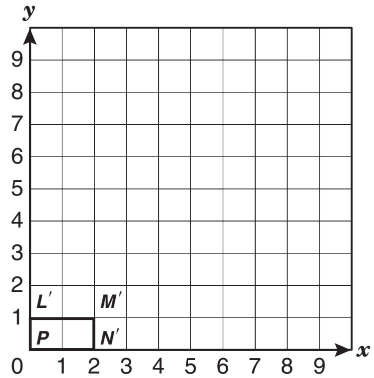
C



B



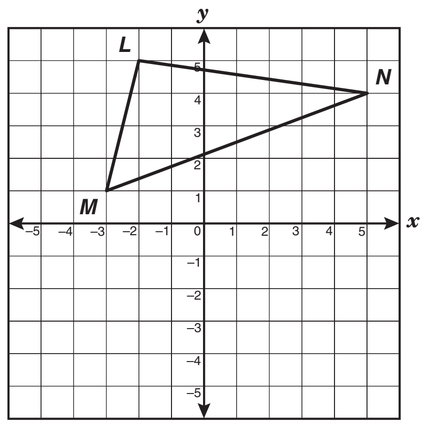
D



Answer Key: page 295

Question 46

If $\triangle LMN$ in the graph below is reflected across the x-axis, what are the coordinates of N'?



A $(-5, 4)$

B $(-4, 5)$

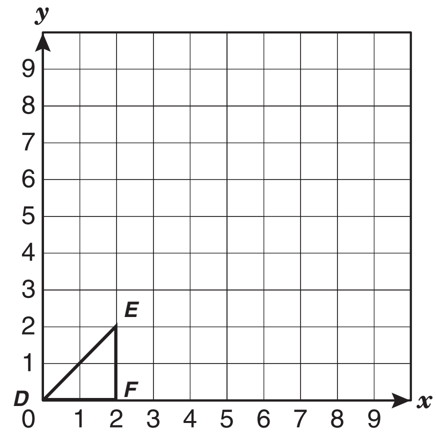
C $(4, -5)$

D $(5, -4)$

Answer Key: page 296

Question 47

$\triangle DEF$ is graphed below.



If $\triangle DEF$ is dilated by a scale factor of 3 using the origin as the center of dilation, what will be the coordinates of E'?

A $(2, 6)$

B $(3, 3)$

C $(6, 2)$

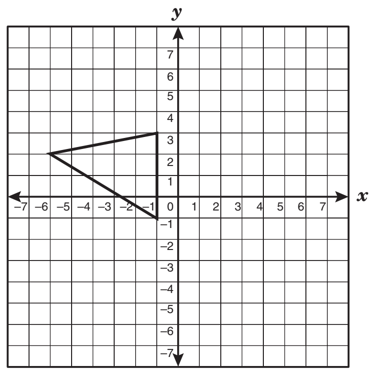
D $(6, 6)$

Answer Key: page 296

MATHEMATICS

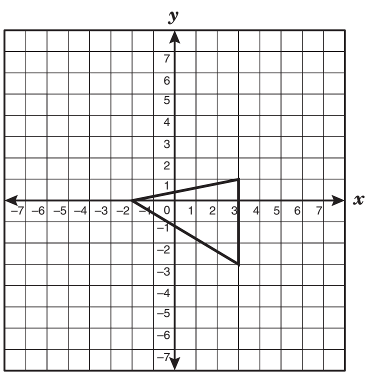
Question 48

The triangle in the graph below will be translated 4 units to the right and 2 units down.

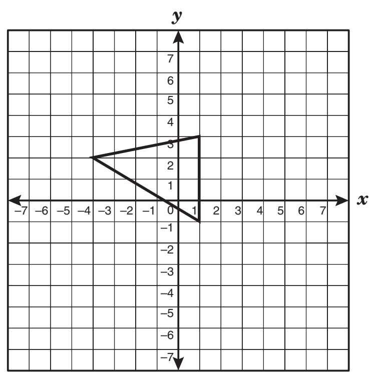


Which graph shows this transformation?

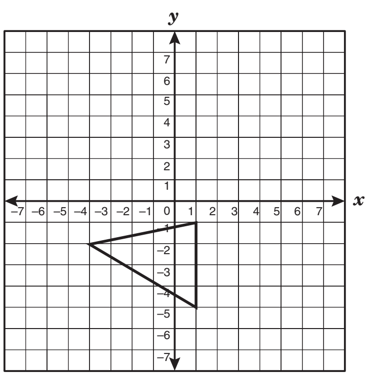
A



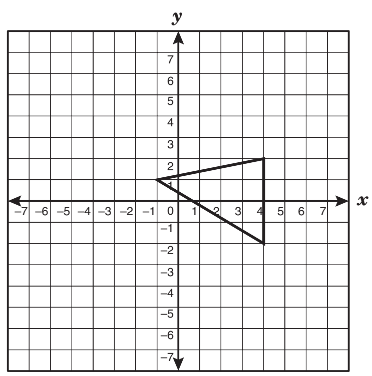
C



B



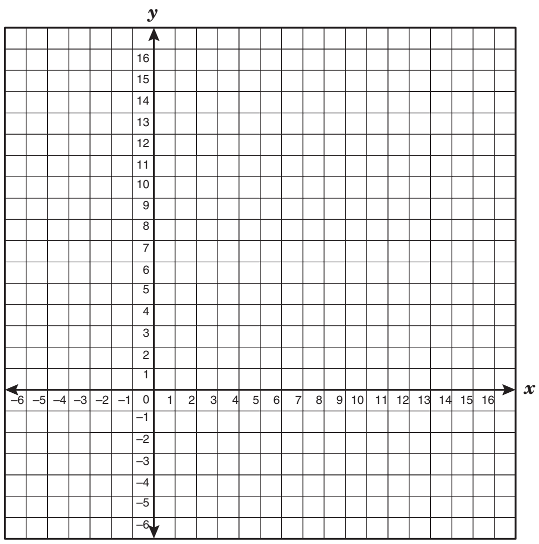
D



Answer Key: page 296

Question 49

The midpoint of $\overline{RS}$ is (8, 5). If one endpoint, point S, has the coordinates (3, −5), what are the coordinates of point R, the other endpoint of $\overline{RS}$?



A (5.5, 0)

B (−3, 5)

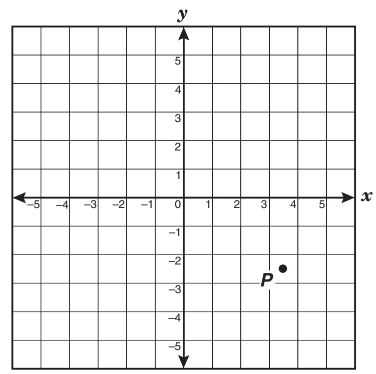
C (11, 0)

D (13, 15)

Answer Key: page 296

Question 50

Which ordered pair best represents the coordinates of point P?



A (−3.5, 4.5)

B (−2.5, 3.5)

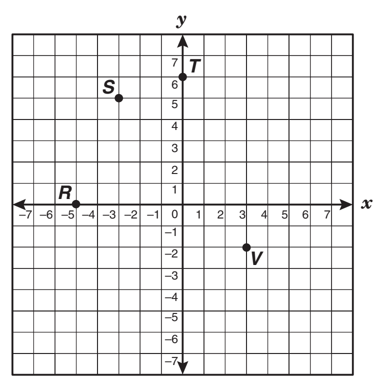
C (3.5, −2.5)

D (4.5, −3.5)

Answer Key: page 296

Question 51

Which of the points on the graph satisfies the conditions $x < -1$ and $y > 2$?



A Point R

B Point S

C Point T

D Point V

Answer Key: page 296

Objective 7

The student will demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.

For this objective you should be able to use geometry to model and describe the physical world.

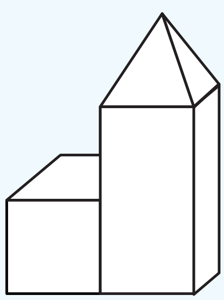
How Do You Recognize a Solid from Different Perspectives?

Given a drawing of a three-dimensional figure, a solid, you should be able to recognize other drawings that represent the same figure from a different perspective.

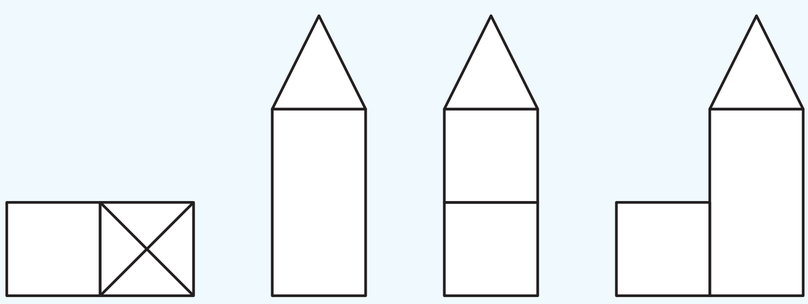
A three-dimensional figure can be represented by drawing the figure from three different views: front, top, and side.

To recognize the solid from different perspectives, you must visualize what the solid would look like if you were seeing it from the front, from above, or from one side.

This three-dimensional model is made up of a cube, a square prism, and a square pyramid.



Which of the four views below is the top view of the solid?



Visualize what the solid would look like if you were seeing it from directly above. You would see the top face of the cube and the top view of the square pyramid. Only the first drawing matches this description.

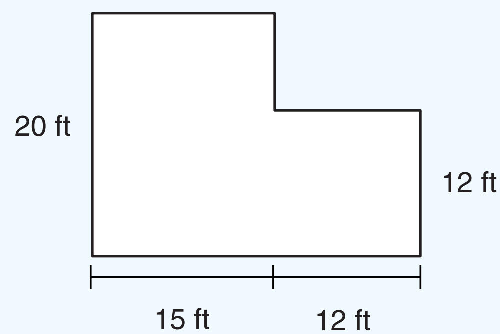
MATHEMATICS

What Kinds of Problems Can You Solve with Geometry?

The laws of geometry govern the physical world around us. You can solve many types of problems using geometry, including problems involving these geometric concepts:

- the area or perimeter of figures;
- the measures of the sides or angles of polygons;
- the surface area and volume of solid figures;
- the ratios of the sides of similar figures; and
- the relationship between the sides of a right triangle.

Yvette has a wooden deck with the dimensions shown below.



She wants to paint the deck with a sealant that will protect the wood. Sealant comes in 1-quart cans. Each can covers 100 square feet. How many cans of sealant will Yvette need to buy in order to seal the entire deck?

Develop a strategy. First find the area of the deck. Then use the area of the deck to determine how many cans of sealant will be needed.

- The deck is composed of a square that is 12 feet by 12 feet and a rectangle that is 20 feet by 15 feet.
- Find the areas of the square and the rectangle.

Square: $A = s^2$
$A = 12^2$
$A = 12 \cdot 12$
$A = 144 \text{ ft}^2$

Rectangle: $A = lw$
$A = 20 \cdot 15$
$A = 300 \text{ ft}^2$

- Add to find the total area of the deck.

$$144 \text{ ft}^2 + 300 \text{ ft}^2 = 444 \text{ ft}^2$$

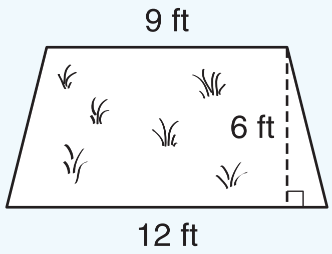
The deck has an area of 444 square feet.

- Divide to find the number of cans of sealant needed.

$$444 \text{ ft}^2 \div 100 \text{ ft}^2 \text{ per can} = 4.44 \text{ cans}$$

Since Yvette cannot buy 4.44 cans of sealant, she will need to buy 5 cans.

Carlie has a plot of grass in the shape of a trapezoid with the dimensions shown below. She wants to plant it with a mixture of wildflower seeds.



If a seed packet that costs $2.95 covers 25 square feet, about how much will it cost Carlie to plant the seeds?

Develop a strategy. First find the area of the plot and then calculate the number of seed packets Carlie will need. Finally, use the number of seed packets to find the cost of the seeds.

- You are given the measures of the bases and the height of the trapezoid-shaped plot of grass. Use these measures to find the area of the plot.

$$A = \frac{1}{2}(b_1 + b_2)h$$
$$A = \frac{1}{2}(12 + 9)6$$
$$A = \frac{1}{2}(21)6$$
$$A = 63 \text{ ft}^2$$

- Divide to find the number of seed packets needed to cover this area.

$$63 \text{ ft}^2 \div 25 \text{ ft}^2 \text{ per packet} = 2.52 \text{ packets}$$

- Carlie needs 2.52 packets. Since she cannot buy 2.52 packets, she must buy 3 packets.

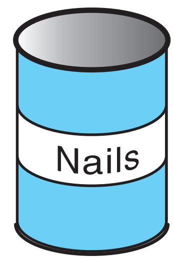
$$\$2.95 \text{ per packet} \times 3 \text{ packets} = \$8.85$$

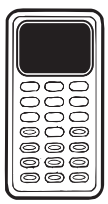
It will cost Carlie $8.85 to plant the wildflower seeds.

MATHEMATICS

Try It

Dave has a workshop in the back of his garage. He uses cans measuring 10 centimeters in diameter to hold items such as nails and screws. Dave wants to wrap tape around each can to label the can's contents.





Dave has 6 cans to label. Will a piece of tape measuring 200 cm in length be long enough to wrap all 6 cans?

Use the formula $C =$ ________________ to find the circumference of 1 can.

The circumference of 1 can is ______ • ______ ≈ ___________ cm.

Dave will need about ___________ centimeters of tape to wrap 1 can.

To wrap 6 cans, he will need 6 • _________ ≈ _________ cm of tape.

A piece of tape measuring 200 centimeters in length is long enough to wrap all 6 cans.

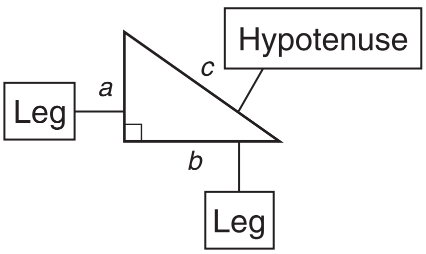
Use the formula $C = \pi d$ to find the circumference of 1 can. The circumference of 1 can is $\pi \cdot 10 \approx 31.4$ cm. Dave will need about 31.4 centimeters of tape to wrap 1 can. To wrap 6 cans, he will need $6 \cdot 31.4 \approx 188.4$ cm of tape. A piece of tape measuring 200 centimeters in length is long enough to wrap all 6 cans.

What Is the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the sides of a right triangle. This special relationship applies only to right triangles.

The sides of a right triangle have special names.

- The **hypotenuse** of a right triangle is the longest side of the triangle. The hypotenuse is always opposite the right angle in the triangle. In the diagram below, the length of the hypotenuse is represented by c.
- The **legs** of the right triangle are the two sides that form the right angle. In the diagram below, the lengths of the legs are represented by a and b.

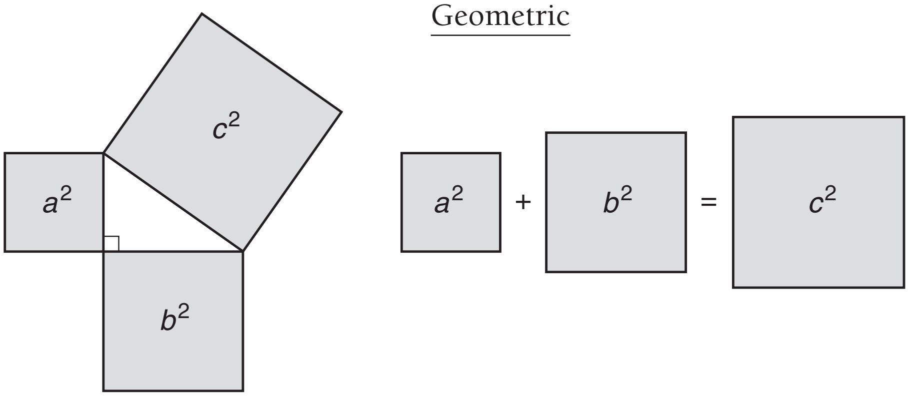


The Pythagorean Theorem can be stated algebraically or verbally.

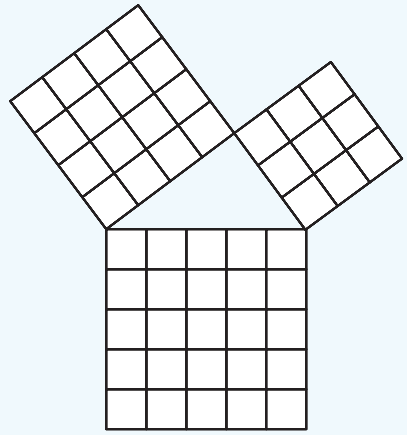
Algebraic	Verbal
In any right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$.	In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

The Pythagorean Theorem can also be interpreted with a geometric model.

Geometric



Does the model below demonstrate the Pythagorean Theorem?



A triangle is a right triangle if its sides satisfy the Pythagorean Theorem. A geometric model of the Pythagorean Theorem shows that the sum of the areas of the squares formed by the legs is equal to the area of the square formed by the hypotenuse.

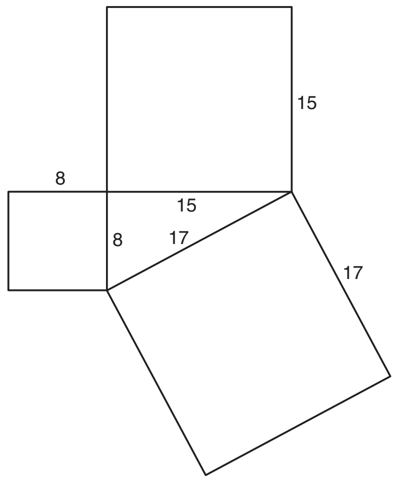
- The square formed by the shorter leg is 3 units by 3 units. It has an area of $3 \cdot 3 = 9$ square units.
- The square formed by the longer leg is 4 units by 4 units. It has an area of $4 \cdot 4 = 16$ square units.
- The square formed by the hypotenuse is 5 units by 5 units. It has an area of $5 \cdot 5 = 25$ square units.

Since $9 + 16 = 25$, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.

This model shows that the sides of the triangle satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$. The sides form a right triangle.

Try It

The model below shows how three squares could be joined to form a triangle. Do the squares form a right triangle?



The longest side is ______ units long, so it is the hypotenuse.

The legs are ______ units and ______ units long.

The square formed by the hypotenuse is 17 units by 17 units.

It has an area of 17 • 17 = ______ square units.

The square formed by the shorter leg is 8 units by 8 units.

It has an area of 8 • 8 = ______ square units.

The square formed by the longer leg is 15 units by 15 units.

It has an area of 15 • 15 = ______ square units.

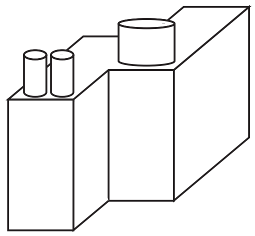
Since ______ = 64 + 225, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs. The triangle is a right triangle.

The longest side is 17 units long, so it is the hypotenuse. The legs are 8 units and 15 units long. The square formed by the hypotenuse is 17 units by 17 units. It has an area of 17 • 17 = 289 square units. The square formed by the shorter leg is 8 units by 8 units. It has an area of 8 • 8 = 64 square units. The square formed by the longer leg is 15 units by 15 units. It has an area of 15 • 15 = 225 square units. Since 289 = 64 + 225, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs. The triangle is a right triangle.

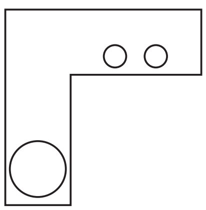
Now practice what you've learned.

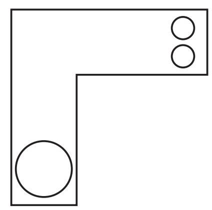
Question 52

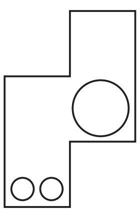
The drawing below shows a 3-dimensional model.

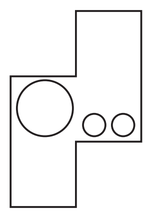


Which best represents the top view of the model?

A


C


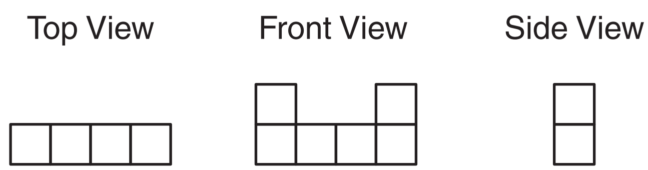
B


D


Answer Key: page 297

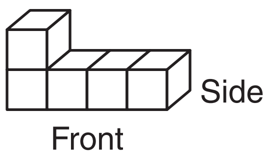
Question 53

The drawings below show the top, front, and side views of a 3-dimensional object.

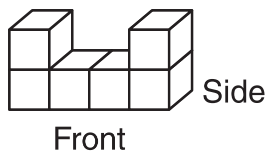


Which solid figure has the top, front, and side views shown?

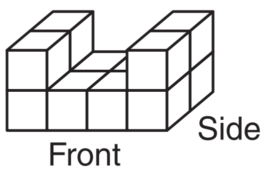
A



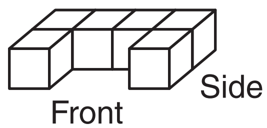
C



B



D



Answer Key: page 297

Question 54

The walls of the building where Tish works have a total area of 18,000 square feet. There are 60 rectangular windows in the building. Each is 6 feet wide by 5 feet tall. What fraction of the building's walls are taken up by windows?

A $\frac{1}{10}$

B $\frac{1}{4}$

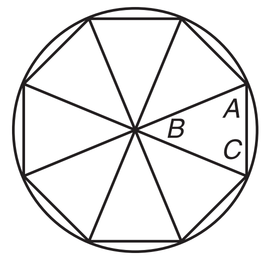
C $\frac{1}{5}$

D $\frac{1}{2}$

Answer Key: page 297

Question 55

A tile pattern is being laid on a patio floor in the shape of a regular octagon inscribed in a circle.



What is the measure in degrees of $\angle A$ in the diagram above?

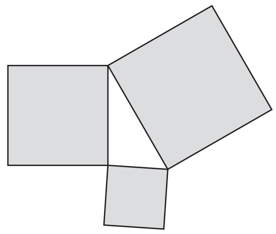
Record your answer and fill in the bubbles. Be sure to use the correct place value.

				.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Answer Key: page 297

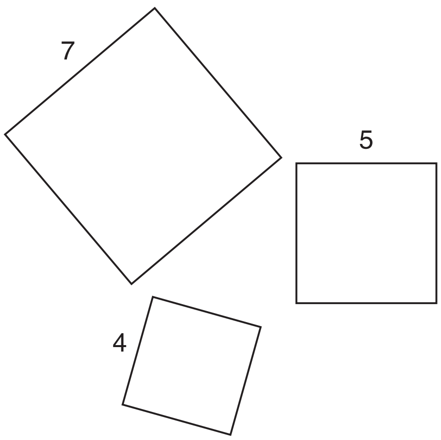
Question 56

The drawing shows how three squares can be joined to form a triangle.

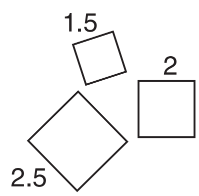


Which set of squares could form a right triangle?

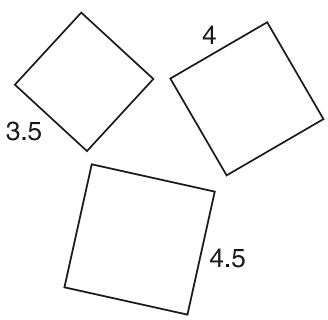
A



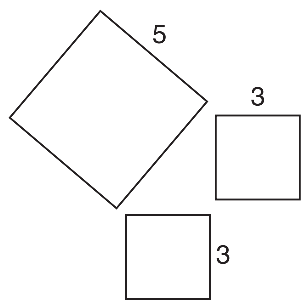
C



B



D



Answer Key: page 298

Question 57

Tran is putting shelf paper on three identical shelves. He buys a roll of shelf paper that is 9 feet long and the same width as the shelves. When he finishes the job, he has a piece of shelf paper left that is 40.5 inches long. What is the length of each shelf?

A 9 in.

B 22.5 in.

C 13.5 in.

D 54 in.

Answer Key: page 298

Objective 8

The student will demonstrate an understanding of the concepts and uses of measurement and similarity.

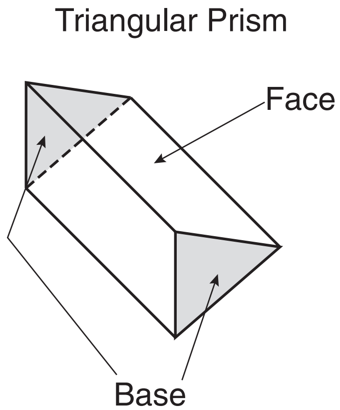
For this objective you should be able to

- use procedures to determine measures of solids;
- use indirect measurement to solve problems; and
- describe how changes in dimensions affect linear, area, and volume measurements.

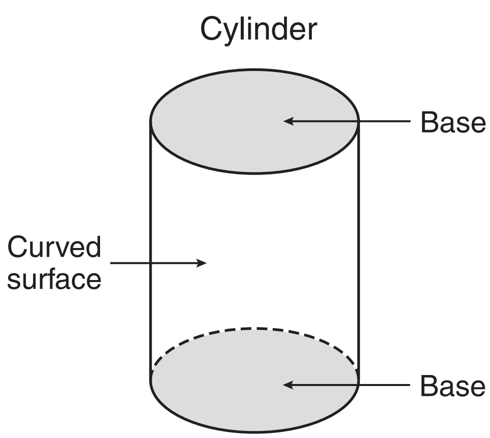
How Do You Find the Surface Area of Solids?

You can use models or formulas to find the surface area of prisms, cylinders, and other solids.

- A **prism** is a solid figure with two bases. The bases are congruent polygons. The other faces of the prism are rectangles. The prism is named by the shape of its bases. For example, a triangular prism has two triangles as its bases.



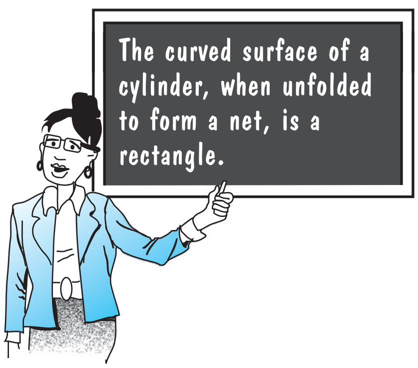
- A **cylinder** is a solid figure with two congruent circular bases and a curved surface.

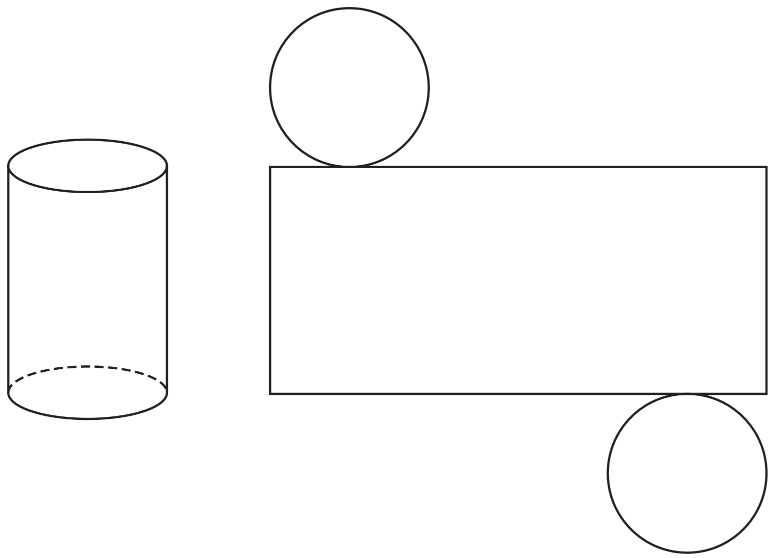


Like the area of a plane figure, the surface area of a solid figure is measured in square units.

- The **total surface area** of a solid figure is equal to the sum of the areas of all its surfaces.
- The **lateral surface area** of a solid figure is equal to the sum of the areas of all its faces and curved surfaces. It does not include the area of the figure's bases.

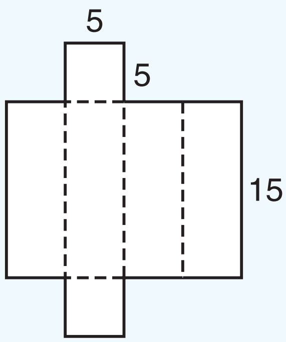
One way to find the surface area of a solid figure is to use a net of the figure. A **net** of a 3-dimensional figure is a 2-dimensional drawing that shows what the figure would look like when opened up and unfolded with all its surfaces laid out flat. Use the net to find the area of each surface.





You can also find the surface area of a solid figure by using a formula. Substitute the appropriate dimensions of the figure into the formula and calculate its surface area. The formulas for the total surface area and lateral surface area of several solid figures are included in the Mathematics Chart.

The net of a square prism is shown below. Find the total surface area of the prism.



The total surface area of the prism is equal to the sum of the areas of all its faces.

Find the area of a square base.

$$A = s^2$$
$$A = 5^2$$
$$A = 25$$

Each square base has an area of 25 square units.

The prism has 2 square bases. Since $2 \cdot 25 = 50$, the combined area of the squares is 50 square units.

Find the area of a rectangular face.

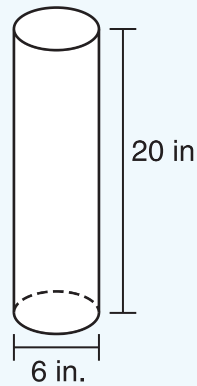
$$A = lw$$
$$A = 5 \cdot 15$$
$$A = 75$$

Each rectangular face has an area of 75 square units.

The prism has 4 rectangular faces. Since $4 \cdot 75 = 300$, the combined area of the rectangles is 300 square units.

The total surface area of the square prism is $50 + 300 = 350$ square units.

What is the lateral surface area of the cylinder shown below?



Use the formula for the lateral surface area of a cylinder, $S = 2\pi rh$.

- The diameter of the cylinder is 6 inches.

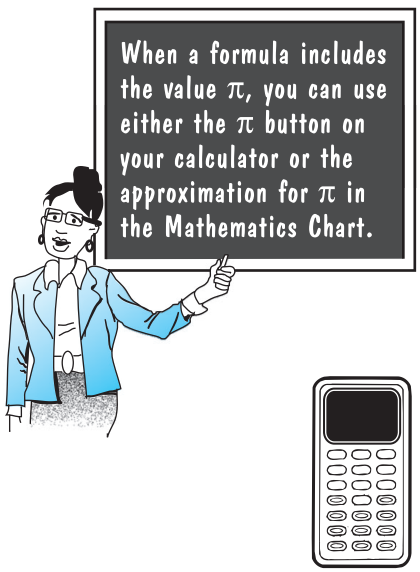
  To find the radius, r, divide the diameter by 2.

  $$r = 6 \div 2$$
  $$r = 3 \text{ in.}$$

- The height, h, of the cylinder is 20 inches.
- Substitute the known values into the formula and solve for the lateral surface area, S.

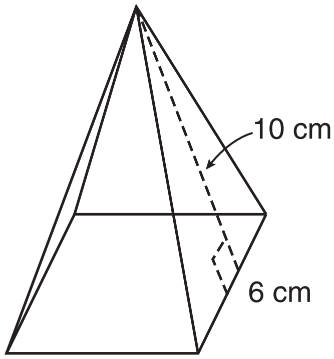
  $$S = 2\pi rh$$
  $$S = 2\pi \cdot 3 \cdot 20$$
  $$S = 2\pi \cdot 60$$
  $$S = 120\pi$$
  $$S \approx 376.99 \text{ in.}^2$$

The lateral surface area of the cylinder is about 377 square inches.



Try It

Quinlan is making a paperweight in the shape shown below. Its base is a square. He wants to cover each of the triangular faces, but not the square base, with gold foil. How many square centimeters of foil will Quinlan need?



The paperweight is shaped like a square ____________________ .

The lateral surface area of the pyramid is equal to the __________ of the areas of its triangular faces; this does not include its square base.

Find the area of a triangular face.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot \_\_\_\_\_ \cdot \_\_\_\_\_$$

$$A = \_\_\_\_\_ \text{ cm}^2$$

Each triangular face has an area of ________ square centimeters.

The pyramid has _____ triangular faces. Since 4 • _____ = _____ , the combined area of the triangular faces is ________ square centimeters. Quinlan will need ________ square centimeters of foil.

The paperweight is shaped like a square pyramid. The lateral surface area of the pyramid is equal to the sum of the areas of its triangular faces; this does not include its square base.

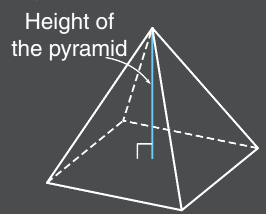
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 6 \cdot 10$$

$$A = 30 \text{ cm}^2$$

Each triangular face has an area of 30 square centimeters. The pyramid has 4 triangular faces. Since 4 • 30 = 120, the combined area of the triangular faces is 120 square centimeters. Quinlan will need 120 square centimeters of foil.

When finding the volume of a prism, cylinder, pyramid, or cone, it is important to remember that the height must be measured along a line perpendicular to the base of the figure—not, for example, along a face of a pyramid.



Do you see that . . .

What Is the Volume of a Solid?

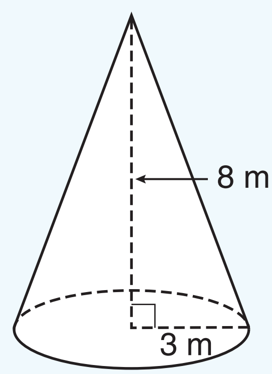
The **volume** of a solid is a measure of the space it occupies. Volume is measured in cubic units.

You can use formulas or models to find the volume of solid figures. The formulas for calculating the volume of several solid figures are in the Mathematics Chart.

When using a formula to find the volume of a solid, follow these guidelines:

- Identify the solid figure you are working with. This will help you select the correct volume formula.
- Use models to help visualize the solid and assign the variables in the volume formula. A model can also be used to find the dimensions of a figure.
- Substitute the appropriate dimensions of the figure for the corresponding variables in the volume formula.
- Calculate the volume. State your answer in cubic units.

Find the volume of the cone shown on the right.



Use the formula for the volume of a cone.

$$V = \frac{1}{3}Bh$$

In this formula B represents the area of the cone's base, and h represents the cone's height.

- Find the area of the base.

  The base of a cone is a circle.

  Use the formula for the area of a circle.

  The circle's radius, r, is 3 meters.

  $$A = \pi r^2 = \pi \cdot 3^2 \approx 28.27 \text{ m}^2$$

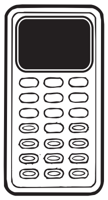
  The area of the cone's base, B, is about 28.27 square meters.
- The cone's height, h, is 8 meters.
- Now substitute the values for B and h into the volume formula.

$$V = \frac{1}{3}Bh \approx \frac{1}{3} \cdot 28.27 \cdot 8 \approx 75.39 \text{ m}^3$$

The volume of the cone is about 75.39 cubic meters.

Try It

An Internet bookstore is shipping a book to a customer. The dimensions of the book are 7 inches by 4 inches by 1.5 inches. The dimensions of the box in which the book will be shipped are 12 inches by 8 inches by 2.75 inches. How many cubic inches of packing material must be added to completely fill the box?



The volume of packing material to be added is equal to the volume of the ____________ minus the volume of the ____________ .

The box and the book are both shaped like a ________________ .

The formula for the volume of a prism is $V =$ ______ .

The bases of a rectangular prism are ______________ . Therefore, the area of a rectangular prism's base, B, is equal to the area of a rectangle ($A = lw$).

Find the volume of the box.

$$V = Bh$$

$$V = lwh$$

$$V = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ \cdot 2.75 = \_\_\_\_\_\_\_\_\_\_ \text{ in.}^3$$

The volume of the box is ______ cubic inches.

Find the volume of the book.

$$V = Bh$$

$$V = lwh$$

$$V = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ \cdot 1.5 = \_\_\_\_\_\_\_\_\_\_ \text{ in.}^3$$

The volume of the book is ______ cubic inches.

Subtract the volume of the book from the volume of the box.

$$\_\_\_\_\_\_ - \_\_\_\_\_\_ = \_\_\_\_\_\_$$

The volume of packing material needed to fill the box is ______ cubic inches.

The volume of packing material to be added is equal to the volume of the box minus the volume of the book. The box and the book are both shaped like a rectangular prism. The formula for the volume of a prism is $V = Bh$. The bases of a rectangular prism are rectangles.

$$V = Bh$$
$$V = lwh$$
$$V = 12 \cdot 8 \cdot 2.75 = 264 \text{ in.}^3$$

The volume of the box is 264 cubic inches.

$$V = Bh$$
$$V = lwh$$
$$V = 7 \cdot 4 \cdot 1.5 = 42 \text{ in.}^3$$

The volume of the book is 42 cubic inches.

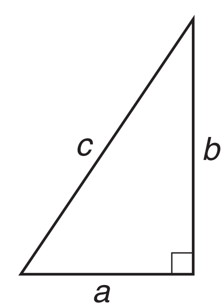
Subtract the volume of the book from the volume of the box.

$$264 - 42 = 222$$

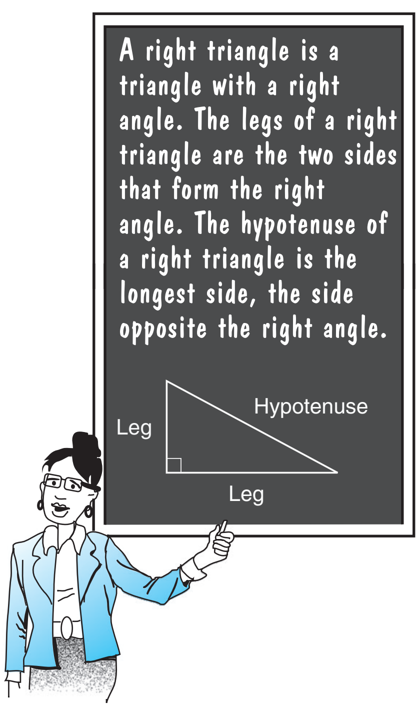
The volume of packing material needed to fill the box is 222 cubic inches.

How Can You Solve Problems Using the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the three sides of a right triangle. The Pythagorean Theorem applies only to right triangles.



- In any right triangle with leg lengths a and b and hypotenuse length c, $a^2 + b^2 = c^2$.
- If the side lengths of any triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle, and c is its hypotenuse.



Any set of three whole numbers that satisfy the Pythagorean Theorem is called a **Pythagorean triple**. The set of numbers {5, 12, 13} forms a Pythagorean triple because these numbers satisfy the Pythagorean Theorem. To show this, substitute 13 for c in the formula—since 13 is the greatest number—and substitute 5 and 12 for a and b.

$$a^2 + b^2 = c^2$$
$$5^2 + 12^2 = 13^2$$
$$25 + 144 = 169$$
$$169 = 169$$

A triangle with side lengths 5, 12, and 13 units is a right triangle.

Any multiple of a Pythagorean triple is also a Pythagorean triple. Since the set of numbers {5, 12, 13} is a Pythagorean triple, the triple formed by multiplying each number in the set by 2, {10, 24, 26}, is also a Pythagorean triple. A triangle with side lengths 10, 24, and 26 units is also a right triangle.

Is a triangle with side lengths of 32 millimeters, 44 millimeters, and 56 millimeters a right triangle?

Determine whether the side lengths satisfy the Pythagorean Theorem. Since 56 is the greatest length, it would be the length of the triangle's hypotenuse. Substitute 56 for *c* in the formula. Substitute 32 and 44 for *a* and *b*, the two legs.

$$a^2 + b^2 = c^2$$

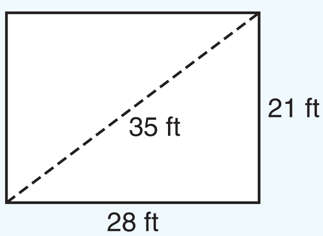
$$32^2 + 44^2 \stackrel{?}{=} 56^2$$

$$1024 + 1936 \stackrel{?}{=} 3136$$

$$2960 \neq 3136$$

Since the side lengths do not satisfy the Pythagorean Theorem, the triangle is not a right triangle.

Allan wanted to enclose an area in his yard with a fence. He marked the widths at 21 feet each and the lengths at 28 feet each. To determine whether his enclosure will form a rectangle, he measured a diagonal and found it to be 35 feet. Is his enclosure a rectangle?



The area Allan enclosed will be a rectangle if its angles are all right angles. The angles are right angles if the two triangles formed by the diagonal are right triangles. Determine whether the dimensions 21 feet, 28 feet, and 35 feet form a right triangle. Substitute 35, the largest value, for *c*.

$$a^2 + b^2 = c^2$$

$$21^2 + 28^2 \stackrel{?}{=} 35^2$$

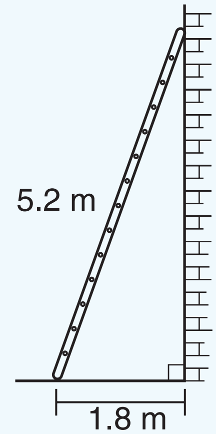
$$441 + 784 \stackrel{?}{=} 1225$$

$$1225 = 1225$$

Since the side lengths satisfy the Pythagorean Theorem, the triangles formed by the diagonal are right triangles. Allan's enclosure will be a rectangle.

MATHEMATICS

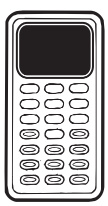
A ladder 5.2 meters in length rests against a wall. If the ladder is 1.8 meters from the base of the wall, how high up the wall does the ladder reach?



- The ladder forms the hypotenuse of the triangle, so let $c = 5.2$ meters.
- The distance from the base of the wall to the ladder forms one leg of the triangle, so let $a = 1.8$ meters.
- The height of the wall from the ground to the top of the ladder is the length of the other leg of the triangle. Let b represent this leg.

Use the Pythagorean Theorem to solve for the length of leg b.

$$a^2 + b^2 = c^2$$
$$(1.8)^2 + b^2 = (5.2)^2$$
$$3.24 + b^2 = 27.04$$
$$b^2 = 23.8$$
$$b = \sqrt{23.8}$$



Find a decimal approximation of $\sqrt{23.8}$.

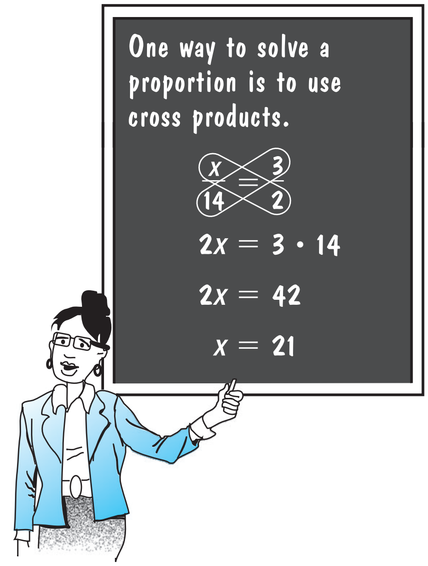
$$\sqrt{23.8} \approx 4.9$$

The ladder reaches about 4.9 meters up the wall.

MATHEMATICS

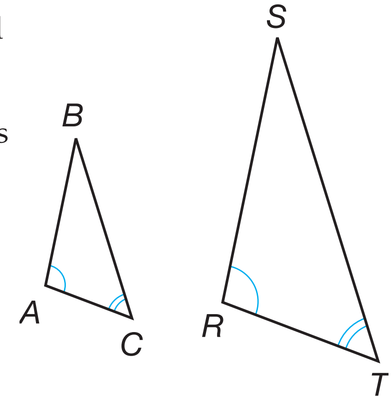
How Can You Use Proportional Relationships to Solve Problems?

You can use proportional relationships to find missing side lengths in similar figures. To solve problems that involve similar figures, follow these guidelines:



- Identify which figures are similar and which sides correspond. Similar figures have the same shape, but not necessarily the same size. The lengths of the corresponding sides of similar figures are proportional.

  Triangle *ABC* is similar to triangle *RST.*

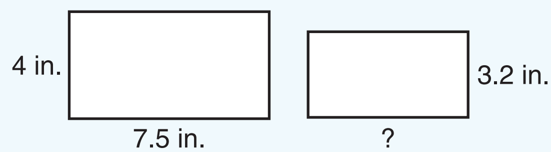


$$\triangle ABC \sim \triangle RST$$

$$\frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}$$

- Write a proportion and solve it.
- Answer the question asked.

The rectangles shown below are similar. Find the length of the smaller rectangle.



- The length of the larger rectangle corresponds to the length of the smaller rectangle, and the width of the larger rectangle corresponds to the width of the smaller rectangle. The ratios of these corresponding sides are equal.

$$\frac{\text{length}_{\text{large}}}{\text{length}_{\text{small}}} = \frac{\text{width}_{\text{large}}}{\text{width}_{\text{small}}}$$

- Substitute the measurements given in the diagram. Let l represent the length of the small rectangle.

$$\frac{7.5}{l} = \frac{4}{3.2}$$

- Use cross products to solve for the length of the smaller rectangle.

$$4l = 7.5 \cdot 3.2$$

$$4l = 24$$

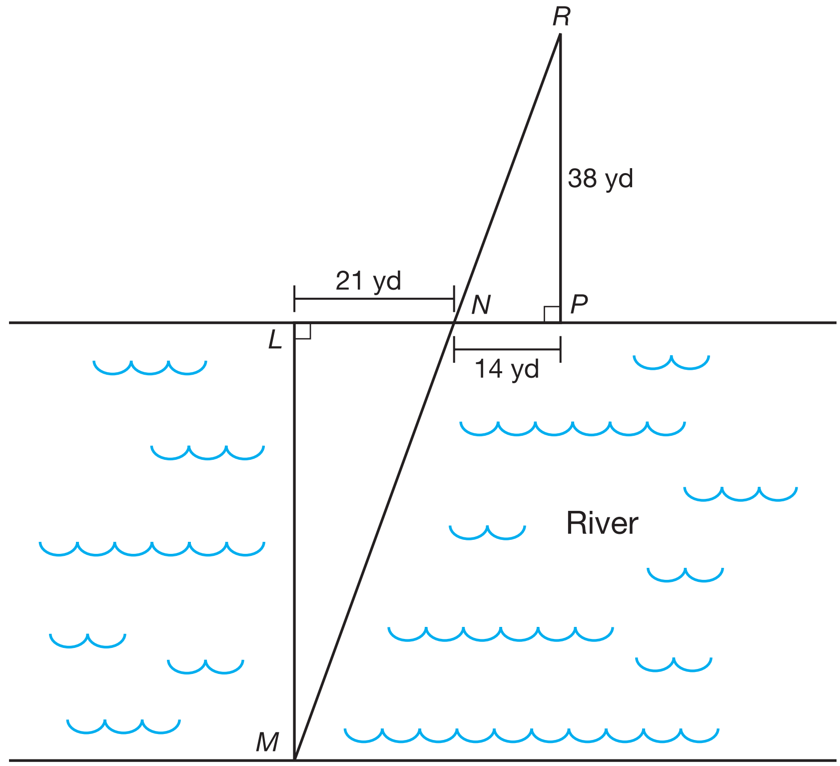
$$l = 6$$

The length of the smaller rectangle is 6 inches.

MATHEMATICS

Try It

A surveyor uses similar triangles to find the distance across a river. He makes the measurements shown in the diagram below.



If $\triangle LMN$ is similar to $\triangle PRN$, what is the width of the river?

In $\triangle LMN$ the length of ______ represents the width of the river.

$\overline{LM}$ corresponds to ______ and $\overline{LN}$ corresponds to ______ .

Since the triangles are similar, the ratios of these corresponding sides are equal.

$$\frac{LM}{\square} = \frac{LN}{\square}$$

Substitute the measurements given in the diagram.

$$\frac{LM}{\square} = \frac{21}{\square}$$

Use cross products to solve for the length of $\overline{LM}$.

$$14 \cdot LM = 38 \cdot 21$$

$$14 \cdot LM = \underline{\hspace{2cm}}$$

$$LM = \underline{\hspace{2cm}} \text{ yd}$$

The width of the river is ________ yards.

In $\triangle LMN$ the length of $\overline{LM}$ represents the width of the river.

$\overline{LM}$ corresponds to $\overline{PR}$ and $\overline{LN}$ corresponds to $\overline{PN}$.

$$\frac{LM}{PR} = \frac{LN}{PN}$$

$$\frac{LM}{38} = \frac{21}{14}$$

$$14 \cdot LM = 38 \cdot 21$$

$$14 \cdot LM = 798$$

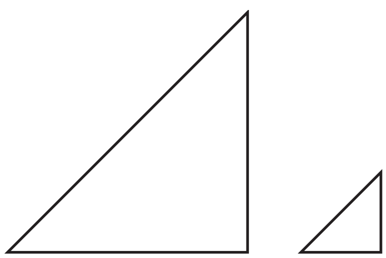
$$LM = 57 \text{ yd}$$

The width of the river is 57 yards.

How Is the Perimeter of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The perimeter of the dilated figure will change by the same scale factor.

To **dilate** a figure means to enlarge or reduce it by a given scale factor. For example, the triangle on the left has been dilated (reduced) by a scale factor of $\frac{1}{3}$ to form the triangle on the right.

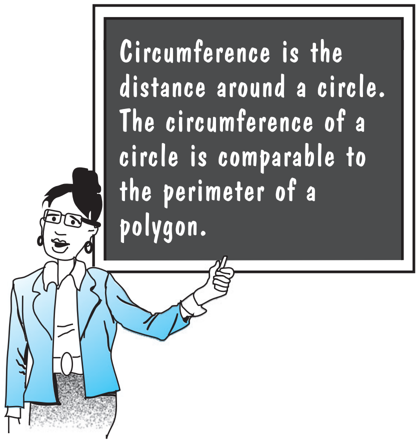


The perimeter of the smaller triangle is $\frac{1}{3}$ the perimeter of the larger triangle.

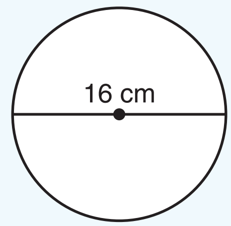
If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their perimeters will be in the ratio $\frac{a}{b}$.

See Objective 6, page 200, for more information about dilations.

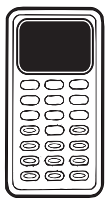
Do you see that . . .



If the circle shown below is dilated by a scale factor of 2, what will be the effect on the circumference of the circle?



The circumference should change by the same scale factor as the dilation. Because the scale factor is 2, the circumference of the circle should increase by a factor of 2.



- To prove this, first find the circumference of the original circle.

$$C = \pi d$$
$$C = \pi \cdot 16$$
$$C \approx 50.27 \text{ cm}$$

- Use the scale factor to find the circumference of the dilated circle.

The circumference of the original circle . . . → $50.27 \cdot 2 = 100.54$ ← . . . should be the circumference of the dilated circle.

. . . times the scale factor . . .

- Use the formula to find the circumference of the dilated circle. The diameter of the dilated circle will be the diameter of the original circle multiplied by the scale factor, 2. The diameter of the dilated circle will therefore be $16 \cdot 2 = 32$ centimeters.

$$C = \pi d$$
$$C = \pi \cdot 32$$
$$C \approx 100.54 \text{ cm}$$

- Compare the two circumferences.

$$\frac{100.54}{50.27} = \frac{2}{1}$$

The circumference of the circle will increase by a factor of 2.

Try It

The perimeter of a rectangle is 36 inches. If the rectangle is reduced by a scale factor of $\frac{1}{4}$, what will be the perimeter of the smaller rectangle?

The perimeter of the original rectangle should decrease by a scale factor of $\frac{\square}{\square}$.

perimeter of smaller rectangle = (perimeter of ________ rectangle) • $\frac{\square}{\square}$

P_{smaller} = ________ • $\frac{\square}{\square}$

P_{smaller} = ________ in.

The perimeter of the smaller rectangle will be ________ inches.

The perimeter of the original rectangle should decrease by a scale factor of $\frac{1}{4}$.

perimeter of smaller rectangle = (perimeter of original rectangle) • $\frac{1}{4}$

$P_{\text{smaller}} = 36 \cdot \frac{1}{4}$

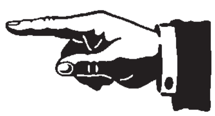
P_{smaller} = 9 in.

The perimeter of the smaller rectangle will be 9 inches.

How Is the Area of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The area of the dilated figure will change by the square of the scale factor.

If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their areas will be in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$.



Do you see that . . .

A triangle with a base of 12 inches and a height of 10 inches is enlarged by a scale factor of 3. How is the area of the triangle affected?

The area of the triangle should increase by the square of the scale factor: $3^2 = 9$. The area should increase by a factor of 9.

- To prove this, first find the area of the original triangle.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 12 \cdot 10$$

$$A = 60 \text{ in.}^2$$

- Use the scale factor to find the area of the larger triangle.

The area of the original triangle . . .

$$60 \cdot 3^2 = 60 \cdot 9 = 540$$

. . . should be the area of the larger triangle.

. . . times the square of the scale factor . . .

- Use the formula to find the area of the larger triangle. The dimensions of the larger triangle are the dimensions of the original triangle multiplied by the scale factor 3.

  Base: $12 \cdot 3 = 36$ in. Height: $10 \cdot 3 = 30$ in.

  Area of the larger triangle:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 36 \cdot 30$$

$$A = 540 \text{ in.}^2$$

- Compare the two areas.

$$\frac{540}{60} = \frac{9}{1} = \left(\frac{3}{1}\right)^2$$

The area of the original triangle increased by a factor of 3^2, or 9.

Try It

An architect draws a floor plan of a building. He uses a scale of 1 inch = 4 feet. The drawing has an area of 150 square inches. What will be the area of the actual building?

Since the dimensions of these two similar figures are in the ratio $\frac{a}{b}$, the areas of the figures are in the ratio $\left(\frac{a}{b}\right)^2 = \frac{\square^2}{\square^2}$.

The dimensions are in the ratio $\frac{\square}{1}$.

The areas should be in the ratio $\left(\frac{\square}{1}\right)^2 = \frac{\square^2}{1^2} = \frac{\square}{1}$.

The area of the drawing is ________ square inches.

150 • ________ = ________

The area of the actual building should be ________ square feet.

Since the dimensions of these two similar figures are in the ratio $\frac{a}{b}$, the areas of the figures are in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$. The dimensions are in the ratio $\frac{4}{1}$. The areas should be in the ratio $\left(\frac{4}{1}\right)^2 = \frac{4^2}{1^2} = \frac{16}{1}$. The area of the drawing is 150 square inches.

150 • 16 = 2400

The area of the actual building should be 2400 square feet.

MATHEMATICS

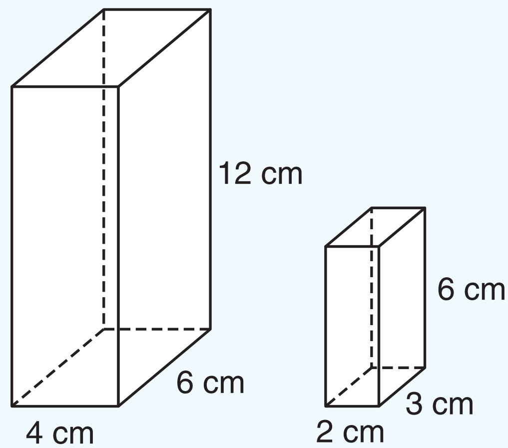
How Is the Volume of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The volume of the dilated figure will change by the cube of the scale factor.

Do you see that . . .

If the dimensions of two similar solid figures are in the ratio $\frac{a}{b}$, then their volumes will be in the ratio $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$.

The dimensions of the prism on the left were reduced by a scale factor of $\frac{1}{2}$ to produce the prism on the right.



By what factor did the volume of the original prism decrease?

The volume of the prism decreased by the cube of the scale factor.

$$\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$$

The volume decreased by a factor of $\frac{1}{8}$.

- To prove this, first find the volume of the original prism.

$$V = Bh$$
$$V = (6 \cdot 4) \cdot 12$$
$$V = 288 \text{ cm}^3$$

- Use the scale factor to find the volume of the dilated prism.

The volume of the original prism . . . → $288 \cdot \left(\frac{1}{2}\right)^3 = 288 \cdot \frac{1}{8} = 36$ ← . . . should be the volume of the dilated prism.

. . . times the cube of the scale factor . . .

- Use the formula to find the volume of the dilated prism.

$$V = Bh$$
$$V = (3 \cdot 2) \cdot 6$$
$$V = 36 \text{ cm}^3$$

The volume of the original prism decreased by a factor of $\frac{36}{288} = \frac{1}{8}$.

Try It

A spherical balloon has a volume of 54 cubic inches. By how many cubic inches would its volume increase if it were further inflated to triple its radius?

If the dimensions of the balloon were increased by a scale factor of ________, then the volume of the balloon would increase by the ____________ of that scale factor.

The volume of the balloon initially is ________ cubic inches.

Its dimensions would increase by a scale factor of ________.

Its volume would increase by a scale factor of ________.

Find the volume of the enlarged balloon.

$$V = \_\_\_\_\_\_\_\_ \cdot (\_\_\_\_\_\_\_\_)^3$$

$$V = \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_$$

$$V = \_\_\_\_\_\_\_\_ \text{ in.}^3$$

The volume of the enlarged balloon would be ________ cubic inches.

The difference between the volumes is 1458 – 54 = ________ cubic inches.

The volume of the balloon would increase by ________ cubic inches.

If the dimensions of the balloon were increased by a scale factor of 3, then the volume of the balloon would increase by the cube of that scale factor. The volume of the balloon initially is 54 cubic inches. Its dimensions would increase by a scale factor of 3. Its volume would increase by a scale factor of $(3)^3$.

$$V = 54 \cdot (3)^3$$

$$V = 54 \cdot 27$$

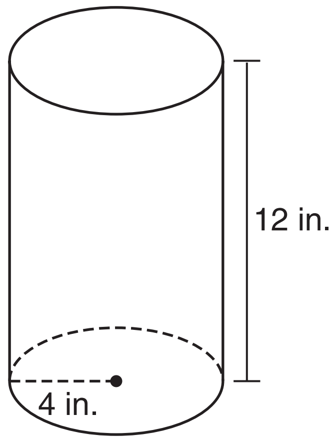
$$V = 1458 \text{ in.}^3$$

The volume of the enlarged balloon would be 1458 cubic inches. The difference between the volumes is 1458 − 54 = 1404 cubic inches. The volume of the balloon would increase by 1404 cubic inches.

Now practice what you've learned.

Question 58

What is the total surface area in square inches of the cylinder shown below?



A 96π in.^{2}

B 128π in.^{2}

C 104π in.^{2}

D 384π in.^{2}

Answer Key: page 298

Question 59

Sterling Coffee is sold in two different cans: regular and large. The dimensions of the large can are twice the dimensions of the regular can. If the regular can has a volume of 72 cubic inches, what is the volume of the large can?

A 144 in.^{3}

B 288 in.^{3}

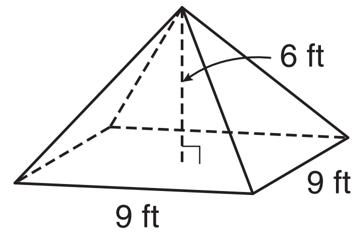
C 576 in.^{3}

D 432 in.^{3}

Answer Key: page 298

Question 60

A square pyramid has the dimensions shown below.



What is the volume of the pyramid in cubic feet?

Record your answer and fill in the bubbles. Be sure to use the correct place value.

				.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Answer Key: page 298

Question 61

A picture frame has a length of 14 inches and a width of 11 inches. A larger frame that is similar in shape has a width of 24 inches. Which measurement is closest to the length of the larger frame?

A 18.9 in.

B 25.0 in.

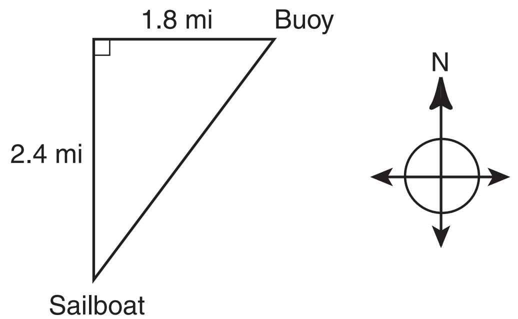
C 30.5 in.

D 27.0 in.

Answer Key: page 299

Question 62

A sailboat is anchored 1.8 miles west and 2.4 miles south of a buoy.



What is the distance between the sailboat and the buoy?

A 9.0 mi

B 4.5 mi

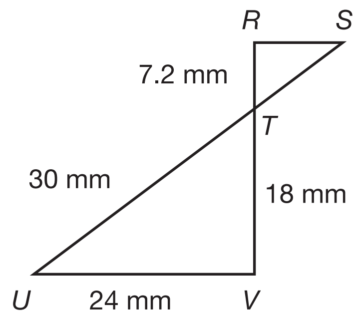
C 4.2 mi

D 3.0 mi

Answer Key: page 299

Question 63

$\triangle RST$ is similar to $\triangle VUT$.



What is the length of $\overline{RS}$?

A 3.3 millimeters

B 9.6 millimeters

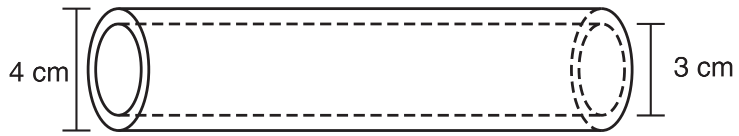
C 9.0 millimeters

D 5.8 millimeters

Answer Key: page 299

Question 64

A water pipe has an outer diameter of 4 centimeters. Its inner diameter is 3 centimeters.



Approximately how many cubic centimeters of water can 50 centimeters of the pipe hold?

A 353 cm^3

B 628 cm^3

C 1413 cm^3

D 2512 cm^3

Answer Key: page 299

MATHEMATICS

Question 65

Carol and Neil both have rectangular vegetable gardens. The area of Carol's garden is 4 times the area of Neil's. Which scale factor represents the ratio of the dimensions of the smaller garden to the dimensions of the larger garden?

A $\frac{1}{2}$

B $\frac{1}{8}$

C $\frac{1}{16}$

D $\frac{1}{4}$

Answer Key: page 300

Question 66

If the lengths of the sides of a square are tripled, how will the perimeter of the new square compare to that of the original square?

A The perimeter will be 3 times the perimeter of the original square.

B The perimeter will be 9 times the perimeter of the original square.

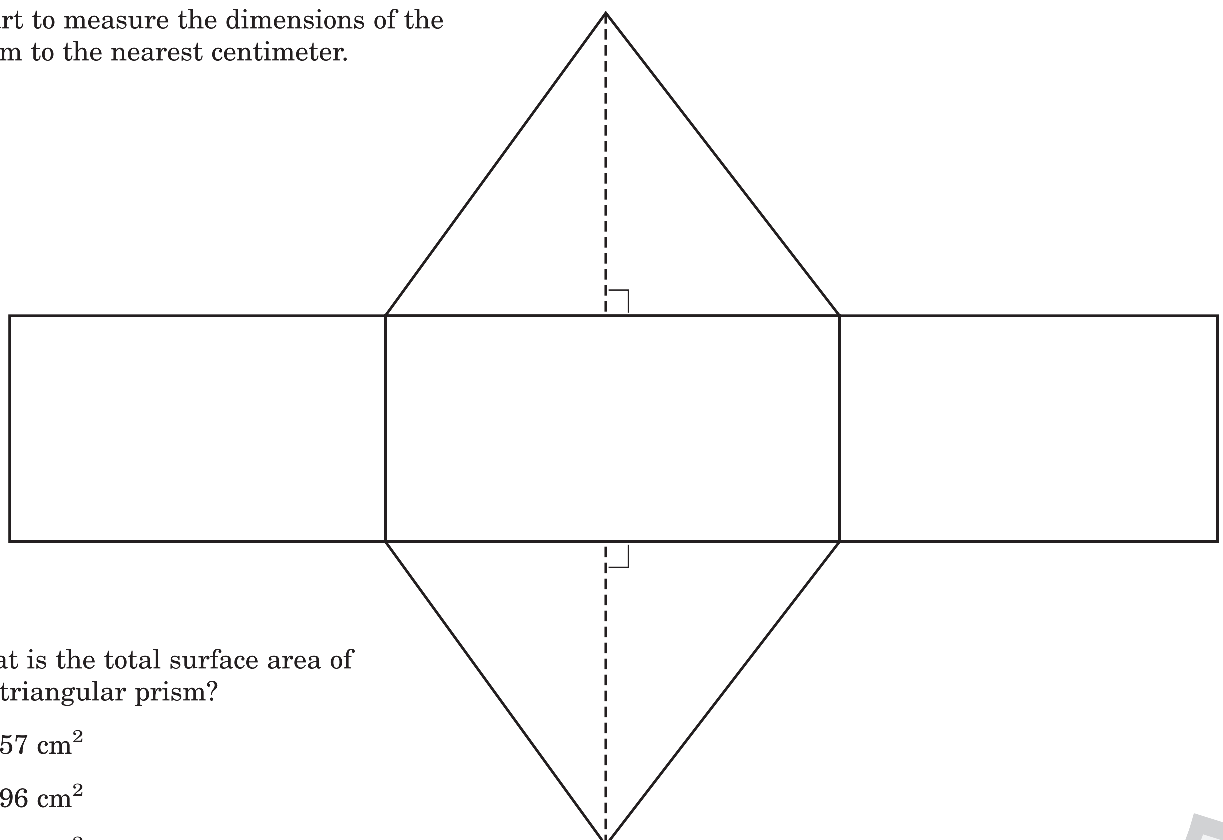
C The perimeter will be 27 times the perimeter of the original square.

D The perimeter will be 6 times the perimeter of the original square.

Answer Key: page 300

Question 67

The net of a triangular prism is shown below. Use the ruler on the Mathematics Chart to measure the dimensions of the prism to the nearest centimeter.



What is the total surface area of the triangular prism?

A 57 cm^2

B 96 cm^2

C 80 cm^2

D 72 cm^2

Answer Key: page 300

Objective 9

The student will demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.

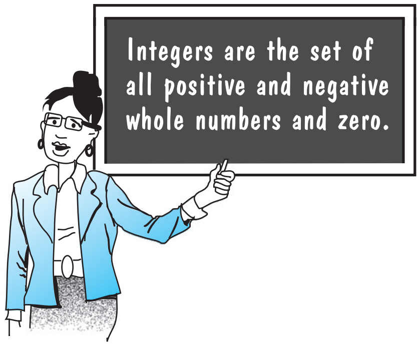
For this objective you should be able to

- understand that different forms of numbers are appropriate for different situations;
- identify proportional relationships in problem situations and solve problems;
- apply concepts of theoretical and experimental probability to make predictions;
- use statistical procedures to describe data; and
- evaluate predictions and conclusions based on statistical data.

What Are Rational Numbers?

Rational numbers are numbers that can be written as the ratio of two integers, $\frac{a}{b}$, where $b \neq 0$. The fraction $\frac{-2}{3}$ is an example of a rational number; it is the ratio of two integers. This rational number could also be written as $-\frac{2}{3}$.

Rational numbers include any real numbers that can be written as fractions. Integers, percents, and some decimals are rational numbers.



Type of Number	Example	Written As a Ratio of Two Integers
An integer	-9	$\frac{-9}{1}$, or $-\frac{9}{1}$
A percent	25%	$\frac{25}{100}$, or $\frac{1}{4}$
A decimal number that terminates or forms a repeating pattern	0.5 $0.\overline{3}$	$\frac{5}{10}$, or $\frac{1}{2}$ $\frac{1}{3}$

MATHEMATICS

How Do You Solve Problems Involving Rational Numbers?

You solve problems that involve rational numbers in the same way you solve any other problem.

- Understand the problem.
- Identify the quantities involved and the relationships between them.
- Write an equation that can be used to find the answer.
- If the numbers in the problem are in different forms, convert them to the same form.
- Solve the equation.
- Check your answer to see whether it is reasonable.

Use these guidelines to convert rational numbers into the same form.

Conversion	Guideline	Example
Fraction to a decimal	Divide the numerator by the denominator.	$\frac{3}{4} = 3 \div 4 = 0.75$
Decimal less than 1 to a fraction	Use the name of the smallest place value—the one farthest to the right—as the denominator. Use the digits to the right of the decimal point as the numerator of the fraction.	$0.35 = \frac{35}{100}$
Decimal greater than 1 to a mixed number	Use the digits to the left of the decimal point as the whole-number part of the mixed number. Convert the digits to the right of the decimal point to a fraction.	$3.28 = 3\frac{28}{100}$
Decimal to a percent	Move the decimal point two places to the right and put a percent sign after the number.	$0.45 = 45\%$
Percent to a decimal	Move the decimal point two places to the left and omit the percent sign.	$3.5\% = 0.035$
Fraction to a percent	First convert the fraction to a decimal. Then convert the decimal to a percent.	$\frac{1}{5} = 0.20 = 20\%$
Percent to a fraction	Express the percent as a fraction with a denominator of 100. If the percent is greater than 100%, it can be expressed as a mixed number.	$35\% = \frac{35}{100}$ $125\% = \frac{125}{100} = 1\frac{25}{100}$

Thomas needs $3\frac{1}{4}$ pounds of ground beef for a recipe. At the grocery store he finds a package of ground beef that weighs 1.56 pounds. How many more pounds of ground beef will he need? Express your answer as a decimal.

To solve this problem, you need to find the difference between the weight needed for the recipe and the weight of the package. These two weights have different forms.

- Express $3\frac{1}{4}$ pounds as a decimal.

$$\frac{1}{4} = 1 \div 4 = 0.25$$

$$3 + \frac{1}{4} = 3 + 0.25 = 3.25$$

The weight needed for the recipe is 3.25 pounds.

- Subtract the weight of the package from the weight needed for the recipe.

$$3.25 - 1.56 = 1.69 \text{ lb}$$

Thomas still needs 1.69 pounds of ground beef for the recipe.

How Do You Solve Problems Involving Proportional Relationships?

A **ratio** is a comparison of two quantities. A **proportion** is a statement that two ratios are equal. There are many real-life problems that involve proportional relationships. For example, you use proportions when converting units of measurement. You also use proportions to solve problems involving percents and rates.

To solve problems that involve proportional relationships, follow these guidelines:

- Identify the ratios to be compared. Be certain to compare the corresponding quantities in the same order.
- Write a proportion, an equation in which the two ratios are set equal to each other.
- Solve the proportion. Use the fact that the cross products in a proportion are equal.

Objective 9

Richard's printer can print 55 pages in 30 minutes. Approximately how many pages can it print in 17 minutes?

- Write a ratio that shows the number of pages printed in 30 minutes.

$$\frac{55 \text{ pages}}{30 \text{ min}}$$

- Write a ratio that shows the number of pages printed in 17 minutes. Let n equal the number of pages.

$$\frac{n \text{ pages}}{17 \text{ min}}$$

- Write a proportion.

$$\frac{55 \text{ pages}}{30 \text{ min}} = \frac{n \text{ pages}}{17 \text{ min}}$$

- Use cross products and solve for n.

$$30n = 55 \cdot 17$$

$$30n = 935$$

$$n \approx 31.2$$

Richard's printer can print approximately 31 pages in 17 minutes.

MATHEMATICS

Try It

Out of 250 students 135 voted for Shannon for class president. What percent of the students voted for Shannon?

A percent is a comparison to ______. Let n equal the number of students out of 100 who voted for Shannon. Write a ratio that shows the number of students out of 100 who voted for Shannon.

$$\frac{n}{\square}$$

Write a ratio that shows the number of students out of 250 who voted for Shannon.

$$\frac{\square}{250}$$

Write a proportion.

$$\frac{n}{\square} = \frac{\square}{250}$$

Use cross products to solve for n.

$$250n = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$$

$$250n = \_\_\_\_\_\_$$

$$n = \_\_\_\_\_\_$$

Therefore, ______ out of 100 students, or ______%, voted for Shannon.

A percent is a comparison to 100. Write a ratio that shows the number of students out of 100 who voted for Shannon: $\frac{n}{100}$. Write a ratio that shows the number of students out of 250 who voted for Shannon: $\frac{135}{250}$.

$$\frac{n}{100} = \frac{135}{250}$$

$$250n = 135 \cdot 100$$

$$250n = 13{,}500$$

$$n = 54$$

Therefore, 54 out of 100 students, or 54%, voted for Shannon.

MATHEMATICS

What Is Probability?

Probability is a measure of how likely an event is to occur. The probability of an event occurring is the ratio of the number of favorable outcomes to the number of all possible outcomes. In a probability experiment favorable outcomes are the outcomes that you are interested in.

The probability, P, of an event occurring must be from 0 to 1.

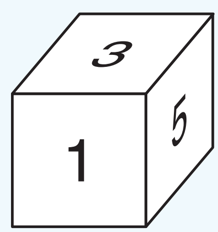
- If an event is impossible, its probability is 0.
- If an event is certain to occur, its probability is 1.

For example, the probability of choosing a pear from a basket containing 2 pears and 3 apples is the ratio of the number of favorable outcomes, 2 pears, to the number of all possible outcomes, 5 pieces of fruit. The probability of choosing a pear is $\frac{2}{5}$.

Do you see that . . .

This is often written as $P(\text{pear}) = \frac{2}{5}$.

A fair number cube has faces numbered 1 through 6. What is the probability of rolling a number greater than 2?



- There are a total of 6 possible outcomes for this experiment: 1, 2, 3, 4, 5, and 6.
- A favorable outcome for this experiment is rolling a number greater than 2. There are 4 favorable outcomes: 3, 4, 5, or 6.
- The probability of rolling a number greater than 2 is the ratio of the number of favorable outcomes to the number of all possible outcomes.

$$\frac{\text{favorable outcomes}}{\text{all possible outcomes}} \quad \frac{4}{6} = \frac{2}{3}$$

The probability of rolling a number greater than 2 is $\frac{2}{3}$.

How Do You Find the Probability of Compound Events?

An event made up of a sequence of simple events is called a **compound event**. For example, flipping a coin and then rolling a die is a compound event.

One way to find the probability of a compound event is to multiply the probabilities of the simple events that make up the compound event.

If $P(A)$ represents the probability of event A and $P(B)$ represents the probability of event B, then the probability of the compound event (A and B) can be represented algebraically.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

For example, the probability of a fair coin landing tails up on one toss is $P(T) = \frac{1}{2}$. The probability of a fair coin landing tails up on two tosses can be found as follows:

$$P(T \text{ and } T) = P(T) \cdot P(T) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

When finding the probability of a compound event, you should first determine whether the simple events included are dependent or independent events.

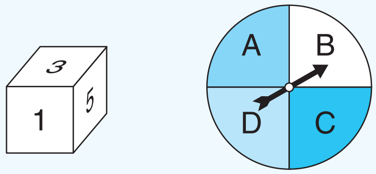
- If the outcome of the first event affects the possible outcomes for the second event, the events are called **dependent events**.

  Suppose you draw 2 marbles, one at a time, from a bag with 4 white marbles and 1 red marble in it and you draw the second marble without replacing the first one you drew. Does the outcome of the first draw affect the likelihood of drawing a red marble on the second draw? Yes, because there were 5 marbles in the bag on the first draw, but only 4 on the second draw. The events are dependent.

- If the outcome of the first event does not affect the possible outcomes for the second event, the events are called **independent events**.

  Suppose you spin a spinner with the numbers 1 through 4 written on it and then spin it again. Does the outcome of the first spin affect the likelihood of spinning a 3 on the second spin? No. The events are independent.

A probability experiment consists of rolling a fair number cube numbered 1 through 6 and then spinning the spinner below.



MATHEMATICS

What is the probability of rolling an even number on the number cube and getting a B on the spinner?

- There are 6 possible outcomes for the number cube: 1, 2, 3, 4, 5, and 6. There are 3 favorable outcomes: rolling a 2, 4, or 6. The probability of rolling an even number on the number cube is $\frac{3}{6} = \frac{1}{2}$.

$$P(\text{even}) = \frac{1}{2}$$

- There are 4 possible outcomes for the spinner: A, B, C, and D. There is only one favorable outcome, B. The probability of getting a B on the spinner is $\frac{1}{4}$.

$$P(\text{B}) = \frac{1}{4}$$

- The probability of rolling an even number on the number cube and getting a B on the spinner is the product of their separate probabilities.

$$\begin{aligned} P(\text{even and B}) &= P(\text{even}) \cdot P(\text{B}) \\ &= \frac{1}{2} \cdot \frac{1}{4} \\ &= \frac{1}{8} \end{aligned}$$

Another way to find the probability of this compound event is to look at the sample space for this experiment and identify the favorable outcomes. This method works only if the outcomes are all equally likely. Use a table to list all the possible outcomes. The three favorable outcomes are shaded.

Sample Space

Number Cube	Spinner	Number Cube	Spinner
1	A	4	A
1	B	**4**	**B**
1	C	4	C
1	D	4	D
2	A	5	A
2	**B**	5	B
2	C	5	C
2	D	5	D
3	A	6	A
3	B	**6**	**B**
3	C	6	C
3	D	6	D

There are 24 possible outcomes; only 3 of them are favorable. The probability of rolling an even number on the number cube and getting a B on the spinner is $\frac{3}{24} = \frac{1}{8}$.

Do you see that . . .

This matches the result obtained using the rule for the probability of a compound event.

$$P(\text{even and B}) = P(\text{even}) \cdot P(\text{B}) = \frac{1}{8}$$

A bag contains 10 red marbles and 12 blue marbles. Once a marble is drawn from the bag, it is not replaced. What is the probability of drawing a red marble and then drawing a blue marble?

The two events are dependent. The outcome of the first draw affects the possible outcomes for the second draw.

- Find the probability of drawing a red marble on the first draw.
  There are 10 favorable outcomes.
  There are 22 possible outcomes.
  $$P(\text{red}_{\text{first}}) = \frac{10}{22} = \frac{5}{11}$$
- Find the probability of drawing a blue marble on the second draw.
  There are 12 favorable outcomes.
  There are now only 21 marbles in the bag, so there are only 21 possible outcomes for the second draw.
  $$P(\text{blue}_{\text{second}}) = \frac{12}{21} = \frac{4}{7}$$

$$\begin{aligned} P(\text{red}_{\text{first}} \text{ and } \text{blue}_{\text{second}}) &= P(\text{red}_{\text{first}}) \cdot P(\text{blue}_{\text{second}}) \\ &= \frac{5}{11} \cdot \frac{4}{7} \\ &= \frac{20}{77} \end{aligned}$$

The probability of drawing a red marble and then a blue marble is $\frac{20}{77}$.

What Is the Difference Between Theoretical and Experimental Probability?

The **theoretical probability** of an event occurring is the ratio comparing the number of ways the favorable outcomes should occur to the number of all possible outcomes. If you toss a coin, theoretically the coin should land on heads $\frac{1}{2}$ of the time.

$$P(H) = \frac{1}{2} = 0.5$$

The **experimental probability** of an event occurring is the ratio comparing the actual number of times the favorable outcome occurs in a series of repeated trials to the total number of trials. If you toss a coin 100 times, it is possible that the coin will land heads up 48 times and tails up 52 times. The experimental probability of the coin landing heads up in this situation would be $\frac{48}{100}$.

$$P(H) = \frac{48}{100} = 0.48$$

The two types of probabilities, theoretical and experimental, are not always equal. In this case the theoretical probability is 0.5, but the experimental probability is 0.48.

Do you see that . . .

For a given situation the experimental probability is usually close to, but slightly different from, the theoretical probability. The greater the number of trials, the closer the experimental probability should be to the theoretical probability.

A number cube has six faces, numbered 1 through 6. Garrett rolls the number cube 50 times and records the results in the frequency table below.

Number Rolled	Frequency
1	6
2	8
3	9
4	8
5	9
6	10

How do the theoretical and experimental probabilities of rolling an even number on the number cube compare?

First determine the theoretical probability of rolling an even number.

- The number of favorable outcomes (2, 4, or 6) is 3.
- The total number of outcomes is 6.
- The theoretical probability of rolling an even number is $\frac{3}{6}$.

$$\frac{3}{6} = \frac{1}{2} = 0.5$$

Next determine the experimental probability of rolling an even number.

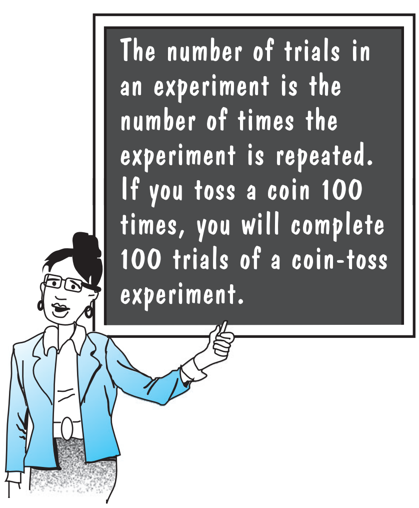
- In the experiment a 2 was rolled 8 times, a 4 was rolled 8 times, and a 6 was rolled 10 times. Add to find the number of times an even number was rolled.

  $$8 + 8 + 10 = 26$$

  An even number was rolled 26 times.
- The total number of trials was 50.
- The experimental probability of rolling an even number is $\frac{26}{50}$.

$$\frac{26}{50} = \frac{13}{25} = 0.52$$

The theoretical probability, 0.5, and the experimental probability, 0.52, are close to each other but not equal.



How Do You Use Probability to Make Predictions and Decisions?

You can use either theoretical or experimental probabilities to make predictions. If you know the probability of an event occurring and you also know the total number of trials, then you can predict the likely number of favorable outcomes.

- Write a ratio that represents the probability of an event occurring.
- Write a ratio that compares the number of favorable outcomes to the number of trials.
- Write a proportion.
- Solve the proportion.

At a clothing factory 130 shirts were inspected. Of these, 4 shirts were found to have defects. Based on this information, predict the number of defective shirts in a batch of 4000 shirts.

First find the experimental probability that a shirt is defective.

- The number of defective shirts in the sample is 4.
- The total number of shirts in the sample is 130.
- The experimental probability that a shirt is defective is $\frac{4}{130} = \frac{2}{65}$.

Next set up and solve a proportion.

- Let n equal the number of defective shirts in a batch of 4000 shirts.
- Write a ratio that shows the number of defective shirts in a batch of 4000 shirts.

$$\frac{n}{4000}$$

- Set this ratio equal to the experimental probability.

$$\frac{n}{4000} = \frac{2}{65}$$

- Use cross products to solve for n.

$$65n = 4000 \cdot 2$$
$$65n = 8000$$
$$n \approx 123$$

In a shipment of 4000 shirts, about 123 will have defects.

Try It

Weather statistics for the last several years show that rain falls on an average of 10 days every November. If this pattern continues, how many rainy days can be expected from November 1 through November 18 this year?

Find the experimental probability of having a rainy day in November.

Out of the 30 days in November, it has typically rained on ______ days. The experimental probability of having a rainy day in November is

$$\frac{\square}{30} = \frac{1}{\square}$$

There are ______ days from November 1 through November 18. Let n represent the number of rainy days in this period.

Write a proportion and solve to find n.

$$\frac{n}{\square} = \frac{\square}{3}$$

$$3n = \underline{\qquad} \cdot \underline{\qquad}$$

$$3n = \underline{\qquad}$$

$$\frac{3n}{\square} = \frac{18}{\square}$$

$$n = \underline{\qquad}$$

The best estimate of the number of rainy days from November 1 through November 18 is ______ days.

Out of the 30 days in November, it has typically rained on 10 days.

The experimental probability of having a rainy day in November is $\frac{10}{30} = \frac{1}{3}$.

There are 18 days from November 1 through November 18.

$$\frac{n}{18} = \frac{1}{3}$$

$$3n = 18 \cdot 1$$

$$3n = 18$$

$$\frac{3n}{3} = \frac{18}{3}$$

$$n = 6$$

The best estimate of the number of rainy days from November 1 through November 18 is 6 days.

How Do You Use Mode, Median, Mean, and Range to Describe Data?

There are many ways to describe the characteristics of a set of data. The mode, median, and mean are all called **measures of central tendency**.

Mode	The **mode** of a set of data describes which value occurs most frequently. If two or more numbers occur the same number of times and more often than all the other numbers in the set, those numbers are all modes for the data set. If each of the numbers in a set occurs the same number of times, the set of data has no mode.	Use the mode to show which value or values in a set of data occur most often. For the set {$\underline{1}$, 4, 9, 3, $\underline{1}$, 6} the mode is 1 because it occurs most frequently. The set {$\underline{1}$, 4, $\underline{3}$, $\underline{3}$, $\underline{1}$, 6} has two modes, 1 and 3, because they both occur twice and most frequently.
Median	The **median** of a set of data describes the middle value when the set is ordered from greatest to least or from least to greatest. If there are an even number of values, the median is the average of the two middle values. Half the values are greater than the median, and half the values are less than the median. The median is a good measure of central tendency to use when a set of data has an outlier, a number that is very different in value from the other numbers in the set.	Use the median to show which number in a set of data is in the middle when the numbers are listed in order. For the set {1, 4, 9, 3, 6} the median is 4 because it is in the middle when the numbers are listed in order: {1, 3, $\underline{4}$, 6, 9}. For the set {1, 4, 9, 3, 1, 6} the median is $\frac{3+4}{2} = 3.5$ because 3 and 4 are in the middle when the numbers are listed in order: {1, 1, $\underline{3}$, $\underline{4}$, 6, 9}. Their values must be averaged to find the median.
Mean	The **mean** of a set of data describes the average of the numbers. To find the mean, add all the numbers and then divide by the number of items in the set. The mean of a set of data can be greatly affected if one of the numbers is an outlier, a number that is very different in value from the other numbers in the set. The mean is a good measure of central tendency to use when a set of data does not have any outliers.	Use the mean to show the numerical average of a set of data. For the set {1, 4, 9, 3, 1, 6} the mean is the sum, 24, divided by the number of items, 6. The mean is $24 \div 6 = 4$.
Range	The **range** of a set of data describes how big a spread there is from the largest value in the set to the smallest value.	Use the range to show how much the numbers vary. For the set {1, 4, 9, 3, 1, 6} the range is $9 - 1 = 8$.

To decide which of these measures to use to describe a set of data, look at the numbers and ask yourself, *What am I trying to show about the data?*

The manager of a sporting-goods store recorded the number and type of basketballs sold over a one-week period.

Type	Price	Number Sold
Mini	\$9.99	4
Street	\$19.50	12
Indoor	\$19.95	15
Pro	\$21.95	10
Autographed	\$39.99	2

Which measure of the data would best describe:

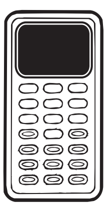
- the average price customers paid for a basketball?
- the price of the most popular type of basketball?

To determine the average price paid for a basketball, the manager would need to calculate the mean of the prices paid.

- Add to find the total number of basketballs sold.

$$4 + 12 + 15 + 10 + 2 = 43$$

- Find the sum of the prices paid and divide by the number sold.



Mean =

$$\frac{4(\$9.99) + 12(\$19.50) + 15(\$19.95) + 10(\$21.95) + 2(\$39.99)}{43} =$$

$$\frac{\$39.96 + \$234.00 + \$299.25 + \$219.50 + \$79.98}{43} =$$

$$\frac{\$872.69}{43} \approx \$20.30$$

The average, or mean, price paid for a basketball was about \$20.30.

To determine the price of the most popular type of basketball, the manager would need to find the price most frequently paid.

A total of 15 indoor basketballs were sold, more than any other type. Therefore, the price the most number of customers paid for a ball was \$19.95, the cost of an indoor ball.

The mode of a set of data is the value that occurs most frequently. The mode of this set of prices is \$19.95, the price of the most popular type of basketball.

Sometimes you need to describe how a change in data affects one or more measures of the data set.

Hal's science class measured the outdoor temperature each weekday at noon for two weeks. The table below shows the data the class recorded.

Outdoor Temperatures (°F)			
1st Week		2nd Week	
Mon.	61	Mon.	55
Tues.	58	Tues.	60
Wed.	55	Wed.	52
Thurs.	64	Thurs.	58
Fri.	59	Fri.	77

On Friday of the second week, the temperature was higher than usual.

How does the higher temperature on Friday affect the different measures of the data?

Range

The range is the difference between the largest value and the smallest value.

- For the first 9 weekdays, the range is $64 - 52 = 12$.
- With the second Friday included, the range is $77 - 52 = 25$.

The range of values increases significantly because 77 is an outlier.

Do you see that . . .

Median

The median is found by listing the temperatures in order and finding the middle value.

- Write the temperature for the first 9 weekdays from least to greatest and find the median temperature.

$$52, 55, 55, 58, 58, 59, 60, 61, 64$$

$$\text{median} = 58$$

- Write the temperatures for all 10 days from least to greatest and find the median temperature.

$$52, 55, 55, 58, 58, 59, 60, 61, 64, 77$$

$$\text{median} = (58 + 59) \div 2 = 58.5$$

The median value is not significantly affected by the outlier.

Mode

The mode of a set of numbers tells which value occurs most frequently.

- For the first 9 weekdays, the modes are 55 and 58.
- With the tenth day included, the modes are still 55 and 58.

The mode is not affected by the outlier.

Mean

The mean is the average of the values.

- Calculate the mean temperature for the first 9 days.

$$\frac{52 + 55 + 55 + 58 + 58 + 59 + 60 + 61 + 64}{9} = 58$$

- Calculate the mean temperature with the 10th day added.

$$\frac{52 + 55 + 55 + 58 + 58 + 59 + 60 + 61 + 64 + 77}{10} = 59.9 \approx 60$$

Since 77 is greater than the other values, it raises the mean.

How Do You Use Graphs to Represent Data?

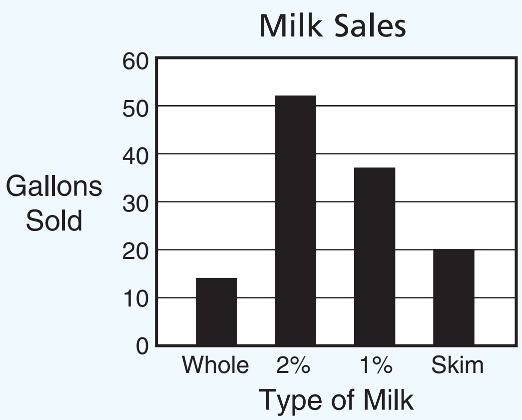
There are many ways to represent data graphically. Bar graphs, histograms, and circle graphs are three types of graphs used to display patterns in data. Graphical representations of data often make it easier to see relationships in the data. However, if the conclusions drawn from a graph are to be valid, you must read and interpret the data from the graph accurately.

A **bar graph** uses bars of different heights or lengths to show the relationships between different groups or categories of data.

The table shows the number of gallons of milk sold at a grocery store during one day. The bar graph represents the same data.

Milk Sales

Type of Milk	Gallons Sold
Whole	14
2%	52
1%	37
Skim	20



What conclusions can you draw from the graph about the number of gallons of milk sold?

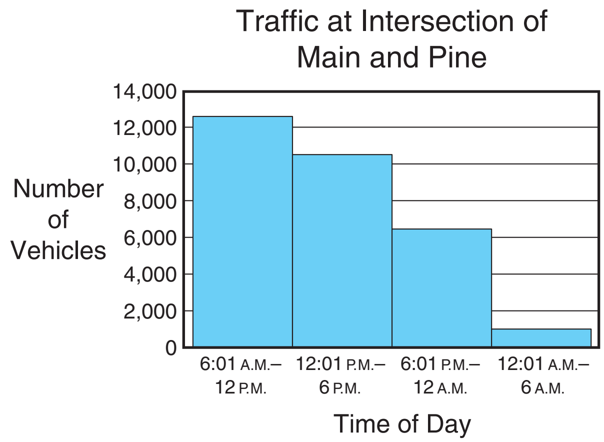
The horizontal axis of the bar graph shows the categories—the different types of milk sold. The vertical axis of the bar graph shows the number of gallons sold of each type of milk.

Conclusions:

- More 2% milk was sold than any other type.
- Whole milk was the least popular type of milk.
- More 2% milk was sold than whole milk and 1% milk combined.

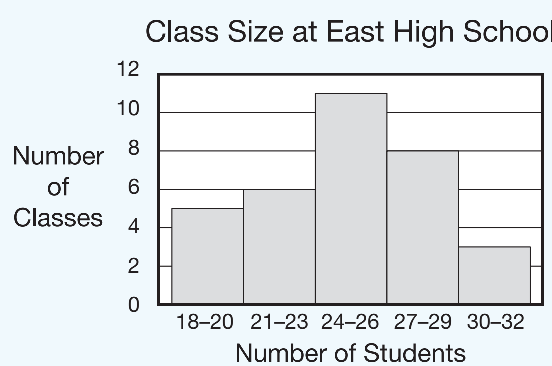
A **histogram** is a special kind of bar graph that shows the number of data points that fall within specific intervals of values. The intervals into which the data's range is divided should be equal. If the intervals are not equal, the graph could be misleading and result in invalid conclusions.

Look at the histogram below showing the number of vehicles passing through an intersection at various times of day.



About 12,400 vehicles passed through the intersection between 6:01 A.M. and 12 P.M. About 10,300 passed through between 12:01 P.M. and 6 P.M. About 6,300 passed through between 6:01 P.M. and 12 A.M. About 1,000 passed through between 12:01 A.M. and 6 A.M.

The histogram below shows the number of classes of different sizes at East High School.



What conclusions can you draw from the graph about the size of the classes at East High School?

- There are no classes with fewer than 18 students.
- The most common class size is from 24 to 26 students.
- There are as many classes with 18 to 23 students as there are classes with 27 to 32 students.

A **circle graph** represents a set of data by showing the relative size of the parts that make up the whole. The circle represents the whole, or the sum of all the data elements. Each section of the circle represents a part of the whole. The number of degrees in the central angle of the section should be proportional to the number of degrees in a circle, 360°.

A poll is conducted to determine the number of students who will vote for each candidate in an upcoming election. The survey results are shown in the table below.

Election Poll

Candidate	Number of Students
Julia	8
Ramón	11
Philippe	15
Roberta	16

What size central angle should be used for the section of the circle graph representing the number of students expected to vote for Ramón?

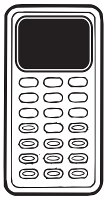
- The table represents a total of 50 students surveyed. Since 11 of them said they would vote for Ramón, the fraction of students who support Ramón is $\frac{11}{50}$.
- Convert $\frac{11}{50}$ to a percent.

$$\frac{11}{50} = \frac{22}{100} = 22\%$$

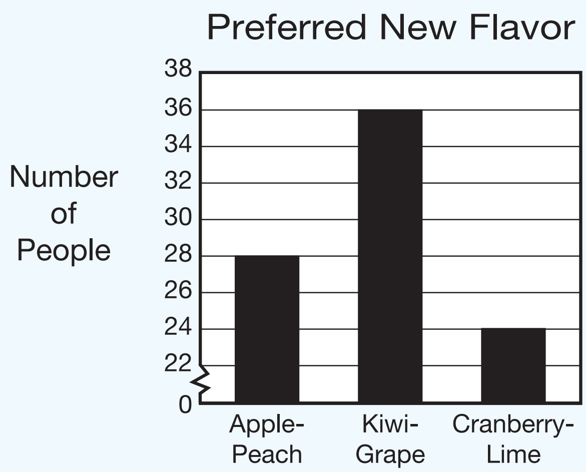
- So, 22% of the students can be expected to vote for Ramón. Therefore, 22% of the circle should be used to represent students who can be expected to vote for Ramón.
- Since a circle has 360°, find 22% of 360°.

$$\frac{22}{100} = \frac{n}{360}$$
$$100n = 7920$$
$$n = 79.2$$

A central angle measuring approximately 79° should be used for the section of the circle graph representing the number of students expected to vote for Ramón.



A juice company conducted a taste test of three new flavors of juice. The results of the taste test are shown in the bar graph below.



An employee of the juice company concluded that more people preferred the flavor kiwi-grape than preferred the other two flavors combined. Is the employee's conclusion valid?

Do you see that . . .

- The length of the bar marked "kiwi-grape" is greater than the sum of the lengths of the bars marked "apple-peach" and "cranberry-lime."

  This may have led the employee to incorrectly conclude that more people preferred the flavor kiwi-grape than preferred the other two flavors combined.

- The broken line on the vertical axis of the graph means that the data from 0 to 22 are not represented on the graph. This is done to make the bars shorter.

- Use the vertical axis to find the number of people surveyed who prefer each flavor.

  apple-peach: 28

  kiwi-grape: 36

  cranberry-lime: 24

- Add to find the total number of people who preferred apple-peach and cranberry-lime combined.

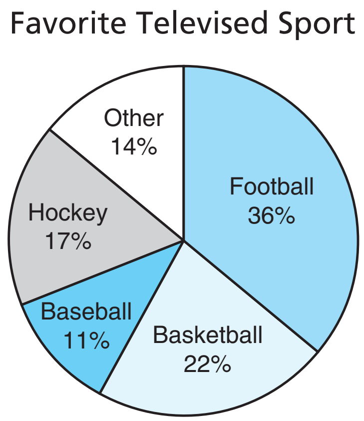
  $$28 + 24 = 52$$

- The number of people who preferred kiwi-grape, 36, is actually less than the number of people who preferred the other two flavors combined, 52.

Therefore, the employee's conclusion is not valid.

Try It

A survey was conducted asking the 300 students at Williams High School to identify their favorite sport to watch on television. The circle graph shows the results of the survey.



After viewing the graph, John concluded that 5 more students in his school prefer to watch basketball than hockey. Was his conclusion valid?

John looked at the graph and saw that

______% of the students prefer to watch basketball and

______% of the students prefer to watch hockey.

From this he should have concluded that ______% more students prefer to watch basketball than hockey.

Find 5% of 300.

$$\frac{\square}{100} = \frac{x}{\square}$$

$$100x = \_\_\_\_ \cdot \_\_\_\_$$

$$100x = \_\_\_\_$$

$$x = \_\_\_\_$$

Since Williams High School has 300 students, that would mean ______ more students prefer to watch basketball than hockey. John's conclusion was not valid.

John looked at the graph and saw that 22% of the students prefer to watch basketball and 17% of the students prefer to watch hockey. From this he should have concluded that 5% more students prefer to watch basketball than prefer to watch hockey.

$$\frac{5}{100} = \frac{x}{300}$$

$$100x = 5 \cdot 300$$

$$100x = 1500$$

$$x = 15$$

Since Williams High School has 300 students, that would mean 15 more students prefer to watch basketball than hockey. John's conclusion was not valid.

Now practice what you've learned.

Question 68

Javier is building a rectangular picture frame $10\frac{1}{4}$ inches by $8\frac{5}{16}$ inches. If Javier uses a 4-foot length of picture molding to make the frame, approximately how many inches will remain on the original length of molding?

A $14\frac{3}{10}$ in.

B $11\frac{1}{8}$ in.

C $11\frac{7}{16}$ in.

D $10\frac{7}{8}$ in.

Answer Key: page 300

Question 69

A skateboard that normally costs $85.00 is on sale for 25% off. What is the sale price of the skateboard?

A $21.25

B $34.00

C $60.00

D $63.75

Answer Key: page 301

Question 70

Janine drove her car 192 miles on 6 gallons of gasoline. At this rate of gasoline usage, how far will she be able to travel on 10 gallons of gasoline?

A 196 mi.

B 320 mi.

C 480 mi.

D 768 mi.

Answer Key: page 301

Question 71

In a box of 20 assorted bagels, 4 bagels contain blueberries, and 5 bagels contain cranberries. What is the probability that a bagel chosen at random from the box contains neither blueberries nor cranberries?

A 5%

B 45%

C 55%

D 95%

Answer Key: page 301

Question 72

A bag contains 6 green apples and 4 red apples. One apple is chosen at random from the bag, and then without replacing the first apple, a second apple is selected. What is the probability of choosing a red apple and then a green apple?

A $\frac{1}{5}$

B $\frac{6}{25}$

C $\frac{4}{15}$

D $\frac{2}{3}$

Answer Key: page 301

Question 73

Luther entered his chili in the cook-off at the county fair. His chili was rated by five judges using a scale of 1–10, in which 1 was the lowest score and 10 the highest. His scores were 3, 3, 6, 7, and 8. Which measure of Luther's data would give his chili the highest final score?

A Mean

B Median

C Mode

D Range

Answer Key: page 301

Question 74

The manager of a video store surveyed a group of customers, asking them the type of movie they were most likely to rent in the future.

Type of Movie	Number
Comedy	12
Drama	20
Action	18

Based on this survey, how many action movies can the store expect to be rented on a day when 425 movies are checked out?

A 139

B 153

C 201

D 360

Answer Key: page 302

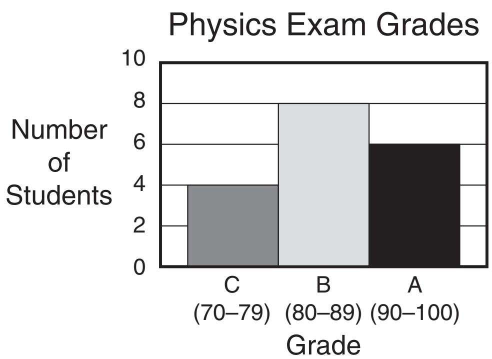
Question 75

The table shows students' grades on a physics exam.

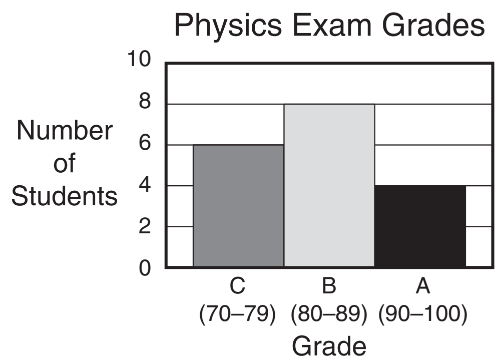
94	92	81	98	88	91
85	91	85	75	90	89
71	80	83	81	77	78

Which histogram correctly represents these data?

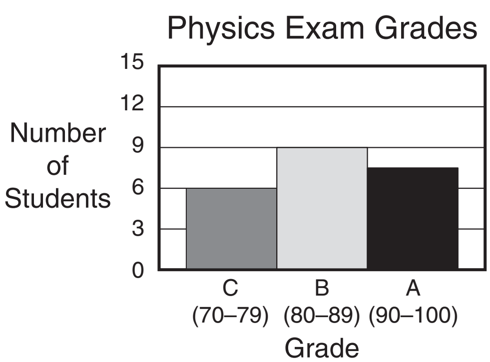
A



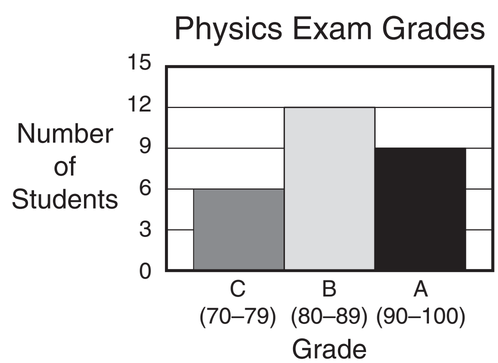
C



B



D



Answer Key: page 302

Question 76

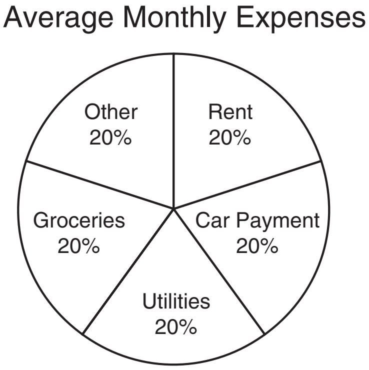
The table below shows Paul's average monthly expenses.

Average Monthly Expenses

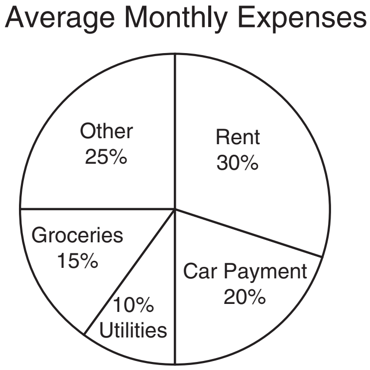
Expense	Amount
Rent	$570
Car payment	$380
Utilities	$190
Groceries	$285
Other	$475

Which circle graph correctly represents the data in the table?

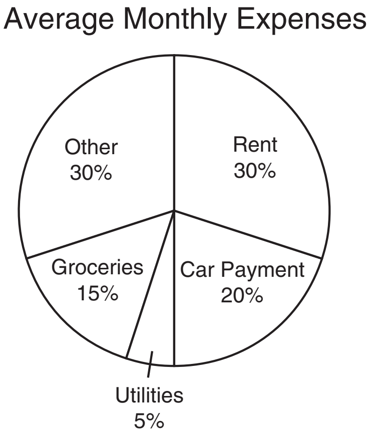
A



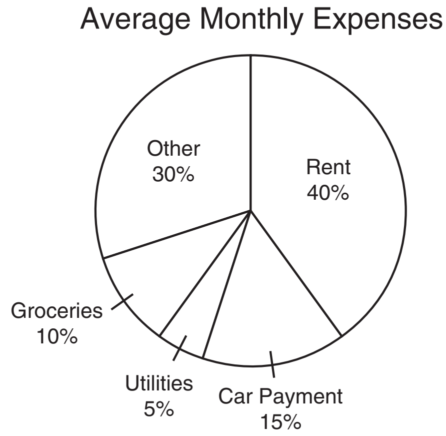
C



B



D

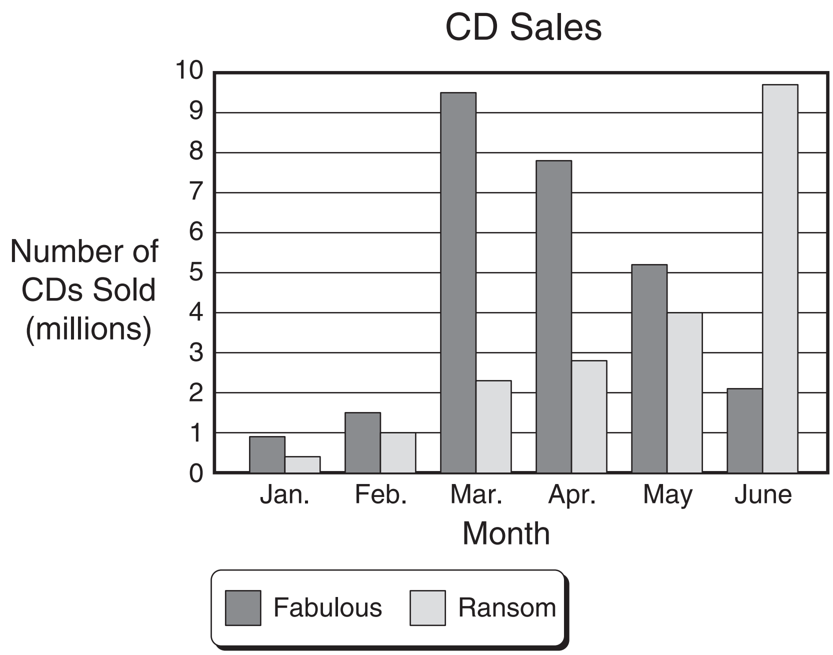


Answer Key: page 302

MATHEMATICS

Question 77

The bar graph below shows the number of CDs two rock groups sold per month over a six-month period.



Which statement is supported by the information in the graph?

A Sales of Fabulous CDs steadily decreased, while sales of Ransom CDs steadily increased.

B Sales of Fabulous CDs remained greater than sales of Ransom CDs throughout the six-month period.

C The range of CD sales over the six-month period is greater for Fabulous than for Ransom.

D The median of the monthly CD sales is greater for Fabulous than for Ransom.

Answer Key: page 302

Objective 10

The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

For this objective you should be able to

- apply mathematics to everyday experiences and activities;
- communicate about mathematics; and
- use logical reasoning.

How Do You Apply Math to Everyday Experiences?

Suppose you want to predict the likelihood of a team winning a game based on its past record. Or suppose you need to find the number of dollars you can earn working at a certain hourly rate. Finding the solution to problems such as these often requires the use of math.

Solving problems involves more than just arithmetic; logical reasoning and careful planning also play very important roles. The steps in problem solving include understanding the problem, making a plan, carrying out the plan, and evaluating the solution to determine whether it is reasonable.

Quentin plans to put new tile on the entire floor of his laundry room. The room has a length of 8 feet, a width of 6 feet, and a height of 7 feet. The tile costs $1.45 per square foot. How much money will Quentin spend on the new tile if he also has to pay a sales tax of 7.5% on the purchase?

- What information is given?

  Dimensions of room: 8 ft long by 6 ft wide by 7 ft high
  Cost of 1 ft^2 of tile: $1.45
  Sales tax rate: 7.5%

- What do you need to find?

  The total cost of the tile, including sales tax

- What information is given but not needed?

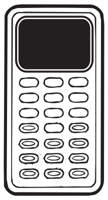
  The height of the room

- To find the area of the floor, use the formula for the area of a rectangle, $A = lw$.

$$A = lw$$
$$A = 8 \cdot 6$$
$$A = 48$$

  The area of the room is 48 square feet.

MATHEMATICS



- To find the cost of the tile, set up a proportion and solve for c, the cost of 48 square feet of tile.

$$\begin{aligned} \frac{c}{48} &= \frac{1.45}{1} \\ 1c &= 48 \cdot 1.45 \\ c &= 69.60 \end{aligned}$$

  The tile costs \$69.60.

- To calculate the sales tax, find 7.5% of the cost of the tile.

$$\begin{aligned} \frac{7.5}{100} &= \frac{x}{69.60} \\ 100x &= 7.5 \cdot 69.60 \\ 100x &= 522 \\ x &= 5.22 \end{aligned}$$

  The sales tax on the purchase is \$5.22.

- To find the total cost, add the sales tax to the cost of the tile.

$$5.22 + 69.60 = 74.82$$

The tile for the laundry room, including sales tax, will cost Quentin \$74.82.

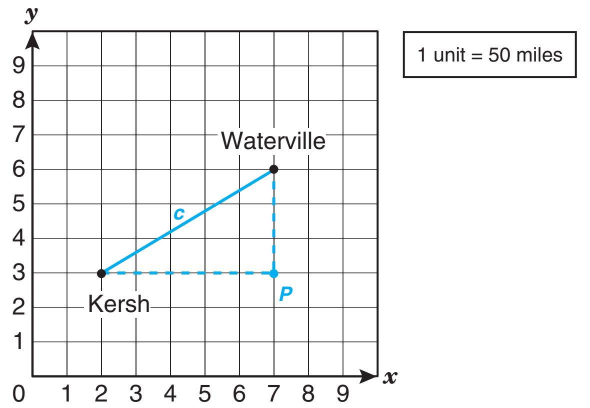
To determine whether \$74.82 is a reasonable answer, divide the total cost (\$74.82 $\approx$ \$75) by the area (48 ft^2 $\approx$ 50 ft^2) and see whether the answer is close to the cost of the tile without tax.

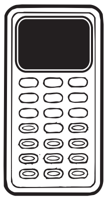
$$\frac{75}{50} = 1.50$$

The answer, \$1.50, is very close to the actual cost of the tile, \$1.45, so the answer makes sense.

Try It

Use the map drawn on the coordinate grid below to find the distance in miles between the towns of Kersh and Waterville.





The map coordinates of Kersh are (_______, _______).

The map coordinates of Waterville are (_______, _______).

The map scale is _______ map unit = _______ miles.

You need to find the distance in _______________ between Kersh and Waterville.

Use the ___________________ Theorem to find the map distance in units between Kersh and Waterville.

Then use the map scale to find the distance in miles between the two towns.

Find the map distance in units between Kersh and Waterville.

Use point P (7, 3) to form a right triangle.

The length of the segment from Kersh to P is _______ units.

The length of the segment from Waterville to P is _______ units.

The length of the segment from Kersh to Waterville is the hypotenuse of the right triangle. Represent this distance with c.

$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + \underline{\qquad\quad}^2$$
$$c^2 = 25 + \underline{\qquad\quad}$$
$$c^2 = \underline{\qquad\quad}$$
$$c = \underline{\qquad\quad}$$

The map distance between the two towns is _________ units.

MATHEMATICS

The map scale is 1 unit = 50 miles. Set up and solve a proportion to find n, the distance in miles between Kersh and Waterville.

$$\frac{n}{\square} = \frac{\square}{1}$$

$$1n = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$$

$$n \approx \_\_\_\_\_\_$$

The distance between the two towns is about ____________________.

The map coordinates of Kersh are (2, 3). The map coordinates of Waterville are (7, 6). The map scale is 1 map unit = 50 miles. You need to find the distance in miles between Kersh and Waterville.

Use the Pythagorean Theorem to find the map distance in units between Kersh and Waterville.

The length of the segment from Kersh to P is 5 units. The length of the segment from Waterville to P is 3 units.

$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 3^2$$
$$c^2 = 25 + 9$$
$$c^2 = 34$$
$$c = \sqrt{34}$$

The map distance between the two towns is $\sqrt{34}$ units.

The map scale is 1 unit = 50 miles. Set up and solve a proportion to find n, the distance in miles between Kersh and Waterville.

$$\frac{n}{\sqrt{34}} = \frac{50}{1}$$

$$1n = \sqrt{34} \cdot 50$$

$$n \approx 291.5$$

The distance between the two towns is about 291.5 miles.

What Is a Problem-Solving Strategy?

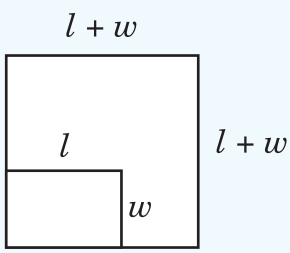
A **problem-solving strategy** is a plan for solving a problem. Different strategies work better for different types of problems. Sometimes you can use more than one strategy to solve a problem. As you practice solving problems, you will discover which strategies you prefer and which work best in various situations.

Some problem-solving strategies include

- drawing a picture;
- looking for a pattern;
- guessing and checking;
- acting it out;
- making a table;
- working a simpler problem; and
- working backwards.

How does the perimeter of a square whose side length is equal to the sum of the length and width of a rectangle compare to the perimeter of the rectangle itself?

To solve this problem, it would be very helpful to draw a picture that shows how the square and the rectangle compare.



One way to solve this problem is to build a table of values by choosing lengths and widths for the rectangle at random, letting the side length of the square equal the sum of the rectangle's length and width, and then looking for a pattern in the perimeters of the rectangle and square.

Rectangle			Square	
Length	Width	Perimeter	Side Length	Perimeter
4	2	$P = 2l + 2w$ $P = 2(4) + 2(2)$ $P = 8 + 4$ $P = 12$	$s = l + w$ $s = 4 + 2$ $s = 6$	$P = 4s$ $P = 4 \cdot 6$ $P = 24$
10	5	$P = 2l + 2w$ $P = 2(10) + 2(5)$ $P = 20 + 10$ $P = 30$	$s = l + w$ $s = 10 + 5$ $s = 15$	$P = 4s$ $P = 4 \cdot 15$ $P = 60$
12	4	$P = 2l + 2w$ $P = 2(12) + 2(4)$ $P = 24 + 8$ $P = 32$	$s = l + w$ $s = 12 + 4$ $s = 16$	$P = 4s$ $P = 4 \cdot 16$ $P = 64$
25	10	$P = 2l + 2w$ $P = 2(25) + 2(10)$ $P = 50 + 20$ $P = 70$	$s = l + w$ $s = 25 + 10$ $s = 35$	$P = 4s$ $P = 4 \cdot 35$ $P = 140$

Look for a pattern.

Perimeter

Rectangle	Square
12	24
30	60
32	64
70	140

The perimeter of the square is always twice the perimeter of the rectangle.

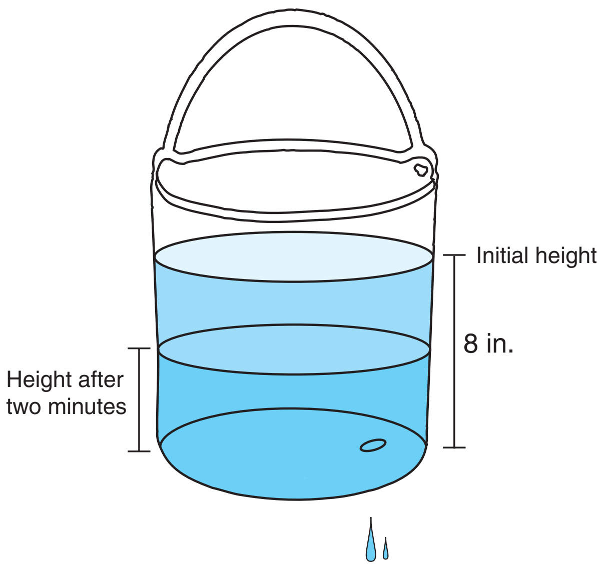
Try It

A plastic pail shaped like a cylinder is filled to a height of 8 inches with rainwater. A hole is punched in the bottom of the pail, causing the water to drain out at a rate of 200 cubic inches per minute. What additional information is needed to determine the number of cubic inches of water remaining in the pail after water drains out for 2 minutes?

To determine what necessary information is missing, you must first identify how you would solve the problem.

Then, as you carry out the problem-solving steps, identify the missing fact.

Draw a picture.



The volume of water that remains in the pail is equal to the original amount of water in the pail ____________ the volume of water that drains out in 2 minutes.

The water drains out of the pail at a rate of ______ cubic inches per minute for 2 minutes.

$$\_\_\_\_\_\_ \cdot 2 = \_\_\_\_\_\_$$

In 2 minutes ______ cubic inches of water drains out of the pail.

The pail is shaped like a ____________. Use the formula for the volume of a cylinder, $V = Bh$, where B is the ____________ of the base and h is the ____________ of the water in the pail.

See Objective 8, page 226, for more information about volume.

MATHEMATICS

Find B. The base of the pail is shaped like a ____________ .

To find the area of the base, use the formula for the area of a circle,

____________ .

You cannot find the area since you do not know the ____________ of the pail.

The ____________ of the pail is the missing piece of information.

The volume of water that remains in the pail is equal to the original amount of water in the pail minus the volume of water that drains out in 2 minutes.

The water drains out of the pail at a rate of 200 cubic inches per minute for 2 minutes.

$$200 \cdot 2 = 400$$

In 2 minutes 400 cubic inches of water drains out of the pail.

The pail is shaped like a cylinder. Use the formula for the volume of a cylinder, $V = Bh$, where B is the area of the base and h is the height of the water in the pail.

The base of the pail is shaped like a circle. To find the area of the base, use the formula for the area of a circle, $A = \pi r^2$. You cannot find the area since you do not know the radius of the pail.

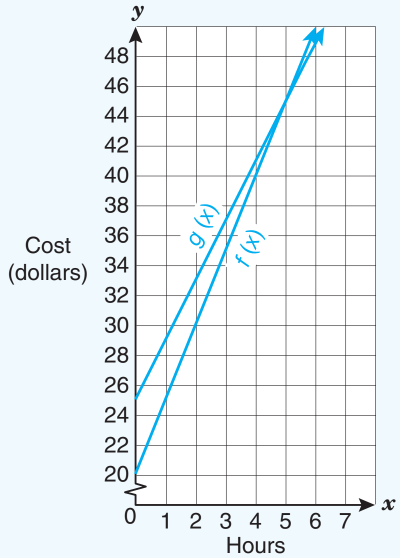
The radius of the pail is the missing piece of information.

How Do You Communicate About Mathematics?

It is important to be able to rewrite a problem using mathematical language and symbols. The words in the problem will give clues about the operations that you will need in order to solve the problem. In some problems it may be necessary to use algebraic symbols to represent quantities and then use equations to express the relationships between the quantities. In other problems you may need to represent the given information using a table or graph.

Orlando plans to rent a canoe. He compares the rental charges at two different shops. The function $f(x) = 5x + 20$ represents the cost of renting a canoe for x hours from Frank's Boat Rental, while $g(x) = 4x + 25$ represents the cost of renting a canoe for x hours from George's Boat Rental. Write a statement that best describes how the cost of renting a canoe from Frank's compares to the cost of renting one from George's.

Draw a graph of the two functions on the same coordinate grid.



Compare the two graphs. As you move from left to right on the x-axis, the number of hours Orlando is renting the canoe increases. Compare the rental costs on the two graphs as the number of rental hours increases.

- For 1 hour it costs \$25 to rent a canoe from Frank's but \$29 to rent a canoe from George's. Frank's is less expensive for 1 hour.
- For 2 hours it costs \$30 to rent a canoe from Frank's but \$33 to rent a canoe from George's. Frank's is less expensive for 2 hours.
- For 5 hours it costs \$45 to rent a canoe from Frank's and \$45 to rent a canoe from George's. The costs are the same for 5 hours.
- For 6 hours it costs \$50 to rent a canoe from Frank's, but \$49 to rent a canoe from George's. George's is less expensive for 6 hours.
- For any number of hours greater than 5, renting a canoe from George's is less expensive.

It costs more to rent a canoe from George's for fewer than 5 hours, but it is less expensive to rent from George's for more than 5 hours.

See Objective 3, page 145, for more information about linear functions.

MATHEMATICS

Can the equation $15x = 330$ be used to solve the problem below?

If 20 times the sum of a number and 5 is 400 more than 5 times the sum of the number and 6, find the number.

- Represent the problem using variables and expressions. Let x equal the unknown number.

  Twenty times the sum of a number and 5 can be represented by $20(x + 5)$.

  Five times the sum of a number and 6 can be represented by $5(x + 6)$.

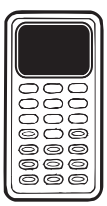
- Write an equation that can be used to find x, if $20(x + 5)$ is 400 more than $5(x + 6)$.

$$20(x + 5) = 5(x + 6) + 400$$

- Simplify the equation.

$$\begin{aligned} 20(x + 5) &= 5(x + 6) + 400 \\ 20x + 100 &= 5x + 30 + 400 \\ 20x + 100 &= 5x + 430 \\ 15x + 100 &= 430 \\ 15x &= 330 \end{aligned}$$

The equation $15x = 330$ can be used to solve the problem.



Try It

A taxi company uses the function $c = 1.25(n - 1) + 3.50$ to determine c, the cost in dollars of a taxi ride in terms of n, the number of miles a rider is transported.

Write a statement that best describes the cost of a ride in terms of the distance traveled.

The cost of a ride begins to be calculated when the number of miles transported is 0, or when $n =$ ______.

$$\begin{aligned} c &= 1.25(n - 1) + 3.50 \\ c &= 1.25(\underline{\hspace{2cm}} - 1) + 3.50 \\ c &= 1.25(\underline{\hspace{2cm}}) + 3.50 \\ c &= \underline{\hspace{2cm}} + 3.50 \\ c &= \underline{\hspace{2cm}} \end{aligned}$$

The cost of a ride begins at \$2.25.

Find the cost of a ride for 1 mile, 2 miles, and 3 miles. Look for a pattern.

Substitute $n = 1$	Substitute $n = 2$	Substitute $n = 3$
$c = 1.25(n - 1) + 3.50$	$c = 1.25(n - 1) + 3.50$	$c = 1.25(n - 1) + 3.50$
$c = 1.25(1 - 1) + 3.50$	$c = 1.25(2 - 1) + 3.50$	$c = 1.25(3 - 1) + 3.50$
$c = 1.25(\_\_\_\_) + 3.50$	$c = 1.25(\_\_\_\_) + 3.50$	$c = 1.25(\_\_\_\_) + 3.50$
$c = \_\_\_\_ + 3.50$	$c = \_\_\_\_ + 3.50$	$c = \_\_\_\_ + 3.50$
$c = \_\_\_\_$	$c = \_\_\_\_$	$c = \_\_\_\_$

The cost of a ride increases at the rate of $____ per mile.

The cost of a taxi ride is $____ plus $____ per mile for each mile transported.

The cost of a ride begins to be calculated when the number of miles transported is 0, or when $n = 0$.

$$c = 1.25(n - 1) + 3.50$$
$$c = 1.25(0 - 1) + 3.50$$
$$c = 1.25(-1) + 3.50$$
$$c = -1.25 + 3.50$$
$$c = 2.25$$

Substitute $n = 1$	Substitute $n = 2$	Substitute $n = 3$
$c = 1.25(n - 1) + 3.50$	$c = 1.25(n - 1) + 3.50$	$c = 1.25(n - 1) + 3.50$
$c = 1.25(1 - 1) + 3.50$	$c = 1.25(2 - 1) + 3.50$	$c = 1.25(3 - 1) + 3.50$
$c = 1.25(0) + 3.50$	$c = 1.25(1) + 3.50$	$c = 1.25(2) + 3.50$
$c = 0 + 3.50$	$c = 1.25 + 3.50$	$c = 2.50 + 3.50$
$c = 3.50$	$c = 4.75$	$c = 6.00$

The cost of a ride increases at the rate of $1.25 per mile. The cost of a taxi ride is $2.25 plus $1.25 per mile for each mile transported.

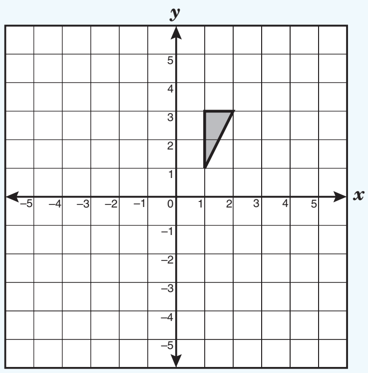
How Do You Use Logical Reasoning as a Problem-Solving Tool?

You can use logical reasoning to find patterns in a set of data. You can then use those patterns to draw conclusions about the data.

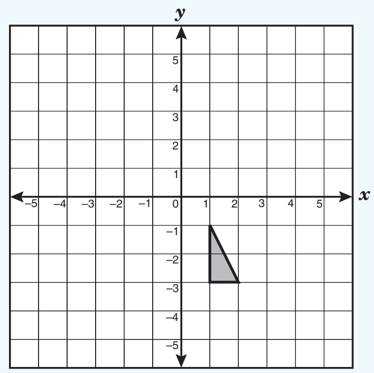
Finding patterns involves identifying characteristics that objects or numbers have in common. Look for the pattern in different ways. A sequence of geometric objects may have some property in common. For example, they may all be rectangular prisms or all be dilations of the same object.

MATHEMATICS

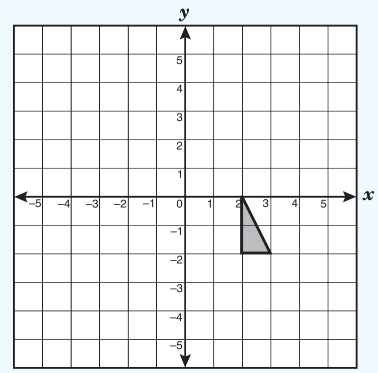
This series of triangles has been formed using transformations. If this pattern continues, identify the coordinates of the sixth triangle.



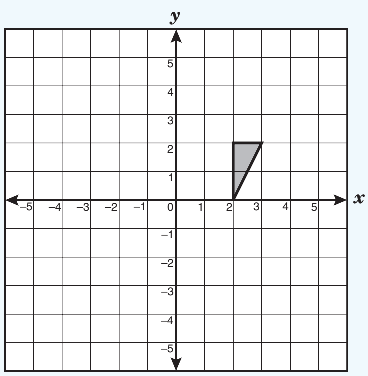
Triangle 1



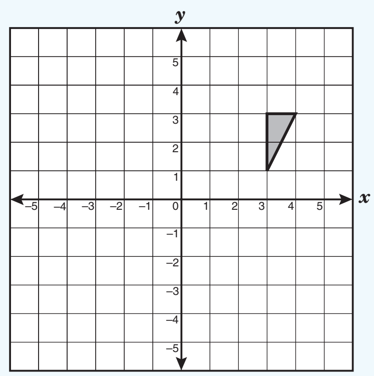
Triangle 2



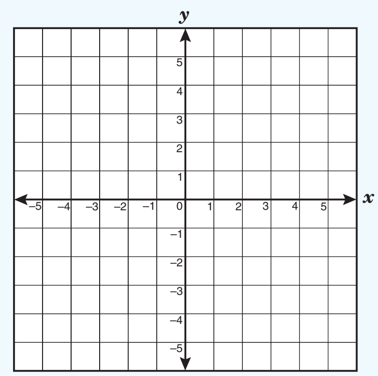
Triangle 3



Triangle 4



Triangle 5



Triangle 6

The triangles have been transformed using a series of reflections and translations.

- The transformation from triangle 1 to triangle 2 is a reflection across the x-axis.
- The transformation from triangle 2 to triangle 3 is a translation of 1 unit right and 1 unit up.
- The transformation from triangle 3 to triangle 4 is a reflection across the x-axis.
- The transformation from triangle 4 to triangle 5 is a translation of 1 unit right and 1 unit up.
- Therefore, the transformation from triangle 5 to triangle 6 should be a reflection across the x-axis.

The coordinates of triangle 5 are (3, 1), (4, 3), and (3, 3). If triangle 5 is reflected across the x-axis, the x-coordinates will not change, but the y-coordinates will be opposite in sign.

The coordinates of triangle 6 will be (3, –1), (4, –3), and (3, –3).

Try It

Over the last five years, a series of surveys were conducted in Williamsville to determine the number of voters who support building a new library in the town. The results of the surveys are shown below.

Survey Data Regarding New Library for Williamsville

	1998	1999	2000	2001	2002
Yes	50	63	92	120	120
No	110	92	88	48	35
Undecided	90	55	50	72	45
Number surveyed	250	210	230	240	200

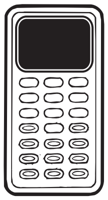
If the pattern of support for a new library continues, how many out of 3500 voters could be expected to vote "Yes" on this issue in 2003?

A different number of people were surveyed each year, so it isn't helpful to compare the number of people surveyed who voted "Yes." There is no obvious pattern to the numbers. Compare the percent of people surveyed each year who supported building a new library.

To find the percent of people surveyed who supported a new library,

______________ the number who voted "Yes" by ________________

______________________________.



Now convert the number who voted "Yes" to an approximate percentage. Complete the table below.

Survey Data Regarding New Library for Williamsville

	1998	1999	2000	2001	2002
Yes	20%		40%		

If the pattern continues, ______% of the voters can be expected to vote "Yes" for a new library in 2003.

Use a proportion to find the number of voters out of 3500 who can be expected to vote "Yes."

$$\frac{\square}{\square} = \frac{x}{3500}$$

$$100x = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$$

$$100x = \_\_\_\_\_\_$$

$$x = \_\_\_\_\_\_$$

In 2003, you can expect ______ voters to vote "Yes."

To find the percent of people surveyed who supported a new library, divide the number who voted “Yes” by the number surveyed.

	1998	1999	2000	2001	2002
Yes	20%	30%	40%	50%	60%

If the pattern continues, 70% of the voters can be expected to vote “Yes” for a new library in 2003.

$$\frac{70}{100} = \frac{x}{3500}$$

$$100x = 70 \cdot 3500$$

$$100x = 245{,}000$$

$$x = 2450$$

In 2003, you can expect 2450 voters to vote “Yes.”

The solution to a problem can be justified by identifying the mathematical properties or relationships that produced the answer. You should have a reason for drawing a conclusion, and you should be able to explain that reason.

The price of a particular stock was \$100 per share on March 1. A stock analyst predicted that the price of the stock would decrease by 10% per month for the next two months. Based on the analyst's prediction, Thomas concluded that the stock price on May 1 should be \$90 per share. Was his conclusion correct?

To determine whether his conclusion was correct, find the amount by which the stock price should decrease between March 1 and April 1. Find 10% of \$100.

$$\frac{10}{100} = \frac{x}{100}$$

$$100x = 1000$$

$$x = 10$$

The stock price should decrease by \$10 per share by April 1. The stock price should be \$100 – \$10 = \$90 per share.

Next find the amount by which the stock price should decrease between April 1 and May 1. Find 10% of \$90.

$$\frac{10}{100} = \frac{x}{90}$$

$$100x = 900$$

$$x = 9$$

The stock price should decrease by \$9 per share. The stock price on May 1 should be \$90 – \$9 = \$81 per share.

Based on the analyst's prediction, Thomas's conclusion of \$90 per share was incorrect. He calculated the decrease in price only for the first month.

Now practice what you've learned.

Question 78

A clerk at an electronics store earns \$7.00 per hour plus an extra \$15.00 for each large-screen television that he sells. The table below shows the number of hours the clerk worked this week.

Day	Hours Worked
Monday	8
Tuesday	7
Wednesday	7.5
Thursday	8
Friday	6

What other information is needed in order to calculate the clerk's earnings for the week?

A The clerk's total amount of sales for the week

B The average cost of a large-screen television

C The number of large-screen televisions the clerk sold

D The pay rate for working more than 40 hours

Answer Key: page 303

Question 79

Julia estimated that it would take her 4 hours to write a 4-page report. It actually took her only 38 minutes to write the first page of the report. If she keeps writing at this same rate, by how many hours and minutes did she overestimate the time it would take her to complete the report?

A 2 hours 32 minutes

B 2 hours 8 minutes

C 2 hours 28 minutes

D 1 hour 28 minutes

Answer Key: page 303

Question 80

Find the linear equation whose slope is 25% greater than the slope of the function $y = 4x + 1$ and whose graph passes through the point (2, 8).

A $y = x + 6$

B $y = 5x - 38$

C $y = 5x - 2$

D $y = x - 6$

Answer Key: page 303

Question 81

A rectangle with dimensions of 3 units by 4 units is enlarged by a scale factor of 1.3. By what percent does its area increase?

A 1.9%

B 69%

C 16.9%

D 9%

Answer Key: page 303

Question 82

There are 16 teams in a basketball tournament. A team is eliminated from the tournament as soon as it loses a game. The tournament ends when there is only one team left that has not lost a game. How many games are played during the tournament?

A 15

B 16

C 30

D 32

Answer Key: page 304

Question 83

The Phillips Sports Company estimates the monthly cost, c, of producing n tennis rackets using the function $c = (9.15 + 4.25)n + 3000$. If the company sells its tennis rackets to retailers for $25 each, how much profit will the company make on the sale of the 500 rackets it produced last month?

A $12,500

B $2,800

C $9,700

D $22,200

Answer Key: page 304

Question 84

Which problem can be solved using the equation below?

$$5x + 65 = 100$$

A Ali bought a pair of shoes for $65 and 5 pairs of socks. If he paid a total of $100, how much did the socks cost per pair?

B Greg lends a friend $100 at a simple interest rate of 5% per year. After how many years will the interest on the loan equal $65?

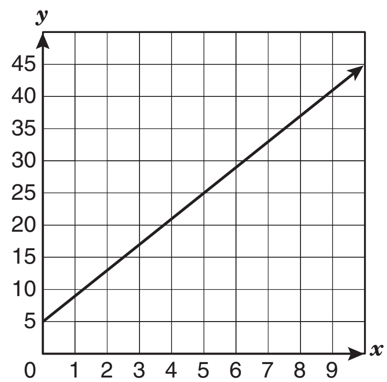
C It took John 5 hours to ride 65 miles during a bike race. If John rode at the same average speed, how long would it take him to ride 100 miles?

D At the beginning of the month, Meg had $100 in her savings account. How much did she have left in her account after a withdrawal of $65 and a deposit of $5?

Answer Key: page 304

Question 85

The graph below best represents which of the following situations?



A Greg earns \$5 per hour for baby-sitting for 4 hours.

B Five students in a classroom each need 4 test tubes.

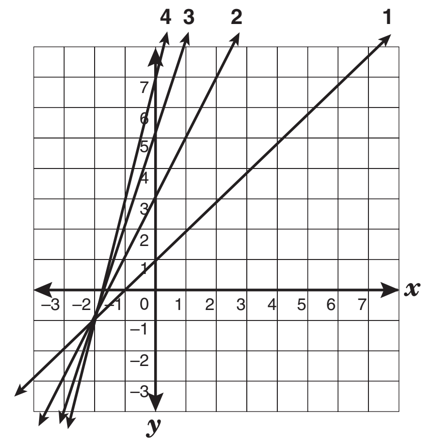
C A bicycle-rental company charges \$5 plus \$4 per hour.

D The length of a garden is 4 feet more than 5 times its width.

Answer Key: page 304

Question 86

A series of equations are graphed on the same coordinate grid.



Which equation's graph would be next in the sequence above?

A $y = x + 9$

B $y = 5x + 8$

C $y = x + 8$

D $y = 5x + 9$

Answer Key: page 305

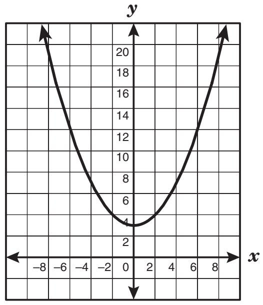
Question 87

Four different functions are presented below. Which functional representation does not belong in the group?

A $f(x) = 2x^2 + 1$

B $2x + 3y = 6$

C



D

x	y
–2	6
0	2
1	3
3	11

Answer Key: page 305

Question 88

A student incorrectly solved the equation $3(2x + 6) - 4 = 14$ as shown below.

Step 1: $3(2x + 6) - 4 = 14$

Step 2: $6x + 6 - 4 = 14$

Step 3: $6x + 2 = 14$

Step 4: $6x = 12$

Step 5: $x = 2$

What mistake did the student make?

A In going from Step 1 to Step 2, the student should have multiplied both terms in parentheses by 3, not just the first term.

B In going from Step 2 to Step 3, the student should have subtracted 4 from the right side of the equation, not the left side.

C In going from Step 3 to Step 4, the student should have added 2 to both sides of the equation instead of subtracting 2.

D In going from Step 4 to Step 5, the student should have multiplied both sides of the equation by 6 instead of dividing by 6.

Answer Key: page 305

MATHEMATICS

Mathematics Answer Key

Objective 1

Question 1 (page 107)

A Correct. The dependent quantity is the quantity that depends on another quantity. The number of miles Jay rides his bike depends on the number of hours he rides it. Therefore, the number of miles Jay rides is the dependent quantity. The number of hours Jay rides is the independent quantity, and the number of miles he can ride his bike in one hour, 12, is a constant.

Question 2 (page 107)

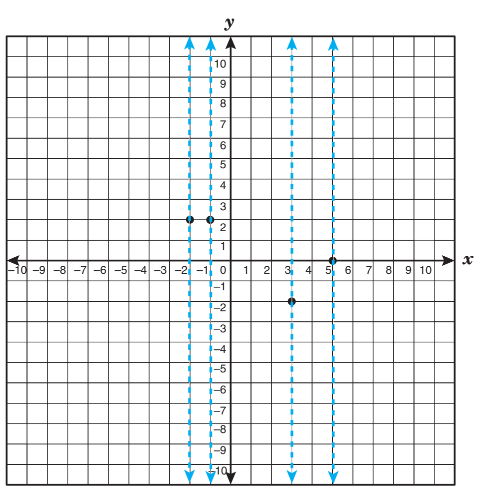
A Correct. The length of the third side of the triangle depends on the length of each of the congruent sides, s, so s is the independent variable in the equation. The value for b depends on the value selected for s. Since b depends on s, it is the dependent variable.

Question 3 (page 107)

B Correct. Check the ordered pairs in the tables to see whether an x-coordinate repeats. The table in choice B contains the ordered pairs (1, 9) and (1, −9). Both ordered pairs contain the x-value 1. The table also contains the ordered pairs (2, 5) and (2, −5). Both ordered pairs contain the x-value 2. In a functional relationship, no x-coordinate should repeat, so this table does not represent a functional relationship.

Question 4 (page 108)

D Correct. Use a vertical-line test to see whether an x-coordinate repeats.



None of the vertical lines intersects more than one point. This graph represents a functional relationship because no x-coordinate repeats.

Question 5 (page 109)

B Correct. The variables in the problem are the total amount of Jean's monthly phone bill, t, and the number of minutes of long-distance calls she uses, m. The total amount of her phone bill is equal to the \$25 basic fee plus long-distance charges. Jean's long-distance charges are equal to the number of long-distance minutes she uses times 7 cents per minute, or $0.07m$. Her monthly phone bill can best be described by the equation $t = 25 + 0.07m$.

Question 6 (page 109)

D Correct. Since the total trip is 182 miles and Rupert has already driven x miles, the number of miles left to drive can be represented by $182 - x$. The time it takes to drive any distance is equal to the distance traveled divided by the rate of speed. If Rupert drove at 70 mph, then $t = \frac{182 - x}{70}$ would represent the amount of time it would take him to complete the drive. But since Rupert will drive slower than 70 mph for the remainder of the trip, it will take him longer than $\frac{182 - x}{70}$ hours to complete the drive. So t must be greater than $\frac{182 - x}{70}$, or $t > \frac{182 - x}{70}$.

Question 7 (page 109)

A Correct.

The graph is a line; so it should be represented by a linear equation. All the answer choices are linear equations. To match the graph to an equation, find the coordinates of at least two points on the graph and check to see whether they satisfy the equation. Two points easily read from the graph are (0, 3) and (−4, 0).

Check (0, 3)	Check (−4, 0)
$y = \frac{3}{4}x + 3$	$y = \frac{3}{4}x + 3$
$3 \stackrel{?}{=} \frac{3}{4}(0) + 3$	$0 \stackrel{?}{=} \frac{3}{4}(-4) + 3$
$3 \stackrel{?}{=} 0 + 3$	$0 \stackrel{?}{=} -3 + 3$
$3 = 3$	$0 = 0$

The equation $y = \frac{3}{4}x + 3$ represents the same function as the graph.

Question 8 (page 110)

C Correct.
To verify that a graph represents a function, find the coordinates of several points on the graph and substitute them into the rule for the function. For example, $(-2, 0)$, $(0, -4)$, and $(2, 0)$ are points on the graph in choice C.

Check to see whether their coordinates satisfy the rule for the function.

Check $(-2, 0)$	Check $(0, -4)$	Check $(2, 0)$
$f(x) = x^2 - 4$	$f(x) = x^2 - 4$	$f(x) = x^2 - 4$
$f(-2) = (-2)^2 - 4$	$f(0) = (0)^2 - 4$	$f(2) = (2)^2 - 4$
$f(-2) = 4 - 4$	$f(0) = 0 - 4$	$f(2) = 4 - 4$
$f(-2) = 0$	$f(0) = -4$	$f(2) = 0$

Yes, the points belong to the function $f(x) = x^2 - 4$. All the points on the graph in choice C satisfy the function rule. The graph in choice C best represents the function $f(x) = x^2 - 4$.

Question 9 (page 111)

C Correct. Look at the graph. In June, 4.4 inches of rain fell. In April and July, 2.5 inches and 2.2 inches of rain fell respectively. A total of 4.7 inches of rain fell in those two months. Less rain, not more, fell in June than in April and July combined.

Question 10 (page 112)

B Correct. Even if Larry doesn't buy any rosebushes from the Garden Club, the cost of joining the club must be shown on the graph. So the Garden Club graph starts at \$25, the membership fee. As Larry buys rosebushes, the total increases by \$10 per bush. The graph for the nursery starts at \$0. As Larry buys rosebushes at the nursery, the total increases by \$15 per bush.

Initially the cost of buying roses through the Garden Club is greater. Its graph is higher. The two graphs intersect when Larry buys 5 rosebushes, which cost him \$75 whether he buys them from the Garden Club or the nursery. After the fifth rosebush, additional rosebushes will cost Larry less through the Garden Club. Its graph is lower.

Objective 2

Question 11 (page 140)

C Correct. The graph is a parabola. Its parent function is the quadratic equation $y = x^2$, whose graph is also a parabola.

Question 12 (page 140)

D Correct. The number of customers who can be checked out, n, depends on the number of cashiers working, x. Therefore, n is the dependent variable. The range is the set of possible values of the dependent variable.

The minimum value for the range would apply if no cashiers were working, or if the function $n = 10x$ was evaluated for $x = 0$.

$$n = 10 \cdot 0 = 0$$

The minimum value for the range is 0.

The maximum value for the range would apply if all the cashiers were working, or if the function $n = 10x$ was evaluated for $x = 20$.

$$n = 10 \cdot 20 = 200$$

The maximum value for the range is 200.

The range of the function is $0 \le n \le 200$.

Question 13 (page 141)

A Correct. The problem states that as rainfall decreases, Jake needs to use more water. So water usage will be greatest in June and August, and it will be least in September.

Question 14 (page 142)

C Correct. As you move from left to right on the x-axis, the data points move down. The data points show that, in general, the more time a student spent on the test, the fewer careless mistakes the student made. Therefore, there is a negative correlation.

Question 15 (page 143)

A Correct. Look at the values in the table and find a pattern. There is a constant difference of 30 seconds between speeches, so the relationship is linear. Look at the number of the week and compare it to the number of seconds the speech lasts. For Week 3 the function is $30 \cdot 3 + 60 = 150$.

Confirm that this pattern fits the other number pairs.

Week	3	4	5	6
Length	150	180	210	240
$30n+60$	$30(3)+60$ $=90+60$ $=150$	$30(4)+60$ $=120+60$ $=180$	$30(5)+60$ $=150+60$ $=210$	$30(6)+60$ $=180+60$ $=240$

The function is $f(n) = 30n + 60$. Since 12 minutes $= 12 \cdot 60 = 720$ seconds, find the number of the week, n, that produces a 720-second speech.

$$\begin{aligned} 30n + 60 &= 720 \\ -60 &= -60 \\ \hline \frac{30n}{30} &= \frac{660}{30} \\ n &= 22 \end{aligned}$$

In Week 22 Eliza will give a 12-minute speech.

Question 16 (page 143)

A Correct. For every ladder rung Brock climbs, he is 1.2 feet higher above the ground. Since Brock is 6 feet tall, the top of his head is already 6 feet from the ground. So 1.2 feet, the height of each rung, times r, the number of rungs, plus 6 feet, Brock's height, represents the distance from the top of Brock's head to the ground. This statement written in mathematical symbols is $1.2r + 6$.

Question 17 (page 143)

C Correct. Test the rule in the table against the values in choice C.

Sequence 9, 13, 17, 21, 25, …

Position Number (n)	$4n + 5$	Value
1	$4(1) + 5$ $= 4 + 5$ $= 9$	9
2	$4(2) + 5$ $= 8 + 5$ $= 13$	13
3	$4(3) + 5$ $= 12 + 5$ $= 17$	17
4	$4(4) + 5$ $= 16 + 5$ $= 21$	21
5	$4(5) + 5$ $= 20 + 5$ $= 25$	25

The rule $4n + 5$ correctly predicts each value in the sequence in choice C.

Question 18 (page 144)

B Correct. One way to find the relationship between the terms in a sequence and their position in the sequence is to build a table. Test the values in the table against the rule in choice B.

Sequence 0, 3, 8, 15, 24, …

Position Number (n)	$n^2 - 1$	Value
1	$1^2 - 1 = 1 - 1 = 0$	0
2	$2^2 - 1 = 4 - 1 = 3$	3
3	$3^2 - 1 = 9 - 1 = 8$	8
4	$4^2 - 1 = 16 - 1 = 15$	15
5	$5^2 - 1 = 25 - 1 = 24$	24

The rule $n^2 - 1$ correctly predicts each value in the sequence.

Question 19 (page 144)

C Correct. Represent the dimensions and area of the original rectangle.

Let the width be represented by w. Since the length is 1 unit greater, let $w + 1$ represent the length.

The area of the original rectangle is equal to its length times its width.

$$A = lw$$

$$A = (w + 1)w = w(w + 1)$$

Represent the dimensions of the larger rectangle by doubling the dimensions of the original rectangle.

Let the width be represented by $2w$. Let the length be represented by $2(w + 1)$, or $2w + 2$.

The area of the larger rectangle is equal to its length times its width.

$$A = 2w(2w + 2)$$

The difference between the areas of the two rectangles is 36 square inches. Subtract the areas to represent the difference. Simplify.

$$\begin{aligned} 2w(2w + 2) - w(w + 1) &= 36 \\ 4w^2 + 4w - w^2 - w &= 36 \\ 4w^2 - 1w^2 + 4w - 1w &= 36 \\ 3w^2 + 3w &= 36 \end{aligned}$$

Question 20 (page 144)

C Correct. To find an expression equivalent to the given expression, you must simplify the given expression.

$$\frac{1}{2}x(4x - 6) + 3(x^2 - 1) =$$

$$\frac{1}{2}x(4x) - \frac{1}{2}x(6) + 3(x^2) - 3(1) =$$

$$\frac{1}{2}(4)(x)(x) - \frac{1}{2}(6)(x) + 3x^2 - 3 =$$

$$2x^2 - 3x + 3x^2 - 3 =$$
$$2x^2 + 3x^2 - 3x - 3 =$$
$$5x^2 - 3x - 3$$

Objective 3

Question 21 (page 170)

A Incorrect. If a person walks for 5 hours at the rate of 10 miles per hour, then the number of miles he walks could best be represented by the equation $y = 5 \cdot 10$.

B Incorrect. If 5 pounds is placed on a scale initially and a series of 10-pound weights are added to it, the total weight on the scale could best be represented by the equation $y = 5 + 10x$.

C Correct. If a waiter is paid \$5 per hour and works for x hours, he will earn $5x$ dollars. If he also earns \$10 in tips, his total wages could best be represented by the equation $y = 5x + 10$.

D Incorrect. The combined length of 5 boards, each 10 feet longer than the width of a doorway, x, could best be represented by the equation $y = 5(x + 10)$.

Question 22 (page 170)

A Correct.
Begin by finding the slope. Choose two points from the table and use the slope formula.

The points (3, 13) and (2, 8) are used here.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - 8}{3 - 2} = \frac{5}{1} = 5$$

Next substitute one point and $m = 5$ into the equation $y = mx + b$.

The point (2, 8) is used here.

$$\begin{aligned} y &= mx + b \\ 8 &= 5(2) + b \\ 8 &= 10 + b \\ -10 &= -10 \\ \hline -2 &= b \end{aligned}$$

Then substitute the values of m and b into $y = mx + b$. The correct equation is $y = 5x - 2$.

Question 23 (page 170)

B Correct. To find the slope, write the equation in slope-intercept form, $y = mx + b$.

$$\begin{aligned} 2x - 5y &= 10 \\ -2x \qquad &= -2x \\ \hline -5y &= -2x + 10 \end{aligned}$$

$$\frac{-5y}{-5} = \frac{-2x}{-5} + \frac{10}{-5}$$

$$y = \frac{2}{5}x - 2$$

In this form of the equation, $m = \frac{2}{5}$. Therefore, the slope of the equation is $\frac{2}{5}$.

Question 24 (page 170)

C Correct. The graph shows a linear relationship between the number of sentences Ryan writes and the time it takes him to write them. Find the slope of the line; it represents the rate at which he writes. At the end of 60 minutes, the graph shows that Ryan has written 90 sentences. Ryan's writing speed is given by the following ratio.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{90}{60} = 1.5$$

Ryan writes at the rate of 1.5 sentences per minute.

Question 25 (page 171)

B Correct.
To find the effect on the graph of the given equation, $6x + 3y = 12$, if it is changed to $6x + 3y = 36$, write the two equations in slope-intercept form, $y = mx + b$, and compare the values of b.

$$\begin{aligned} 6x + 3y &= 12 \\ -6x \qquad &= -6x \\ \hline 3y &= -6x + 12 \\ \frac{3y}{3} &= \frac{-6x}{3} + \frac{12}{3} \\ y &= -2x + 4 \end{aligned}$$

$$\begin{aligned} 6x + 3y &= 36 \\ -6x \qquad &= -6x \\ \hline 3y &= -6x + 36 \\ \frac{3y}{3} &= \frac{-6x}{3} + \frac{36}{3} \\ y &= -2x + 12 \end{aligned}$$

The first line has a y-intercept of 4; the second line has a y-intercept of 12. The graph has been translated up 8 units.

Question 26 (page 171)

A Correct.
Both graphs are lines. To determine their relationship, write their equations in slope-intercept form, $y = mx + b$, and compare the slopes of the graphs. The first equation, $y = \frac{2}{3}x - 4$, is already in that form. Its slope is the value of m, $\frac{2}{3}$.

Transform the second equation.

$$\begin{array}{r} 3x + 2y = 12 \\ \underline{-3x \qquad = -3x} \end{array}$$

$$2y = -3x + 12$$

$$\frac{2y}{2} = \frac{-3x}{2} + \frac{12}{2}$$

$$y = -\frac{3}{2}x + 6$$

The slope of the second line is $-\frac{3}{2}$. The slope of the first line is $\frac{2}{3}$. Since these values are negative reciprocals, the lines are perpendicular.

Question 27 (page 171)

B Correct.

All the answer choices are written in slope-intercept form, $y = mx + b$. Find the values for the slope, m, and the y-intercept, b.

Use the slope formula to find the slope of the line that passes through the points (7, 5) and (11, 9).

$$m = \frac{9-5}{11-7} = \frac{4}{4} = 1$$

The slope of the line is 1.

Find b. Substitute 1 for m and use the coordinates of the point (7, 5) for x and y into the slope-intercept form of the equation of a line.

$$y = mx + b$$
$$5 = 1(7) + b$$
$$5 = 7 + b$$
$$b = -2$$

Substitute $m = 1$ and $b = -2$ into the slope-intercept form of the equation.

$$y = 1x - 2$$

Since $1x$ and x are equivalent, this is the same as choice B, $y = x - 2$.

Question 28 (page 171)

B Correct.

All the answer choices are written in slope-intercept form, $y = mx + b$. Find the values for the slope, m, and the y-intercept, b.

If the y-intercept is 5, then the value of b is 5. Find m.

If the x-intercept is -3, then the point $(-3, 0)$ is a point on the line. Substitute $b = 5$, $x = -3$, and $y = 0$ into the equation $y = mx + b$ to find m.

$$y = mx + b$$
$$0 = m(-3) + 5$$
$$0 = -3m + 5$$
$$-5 = -3m$$
$$\frac{-5}{-3} = \frac{-3m}{-3}$$
$$\frac{5}{3} = m$$

The equation of the line is $y = \frac{5}{3}x + 5$.

Question 29 (page 171)

A Correct. In the graph Mark and his friends have 0 cookies when the time is equal to 0 hours, so the y-intercept is 0. If they have 20 cookies before they start baking (when the time is equal to 0 hours), the y-intercept will increase from 0 to 20. Since the rate at which they bake does not change, the slope would not change. The new graph would be parallel to and higher than the old one. The y-intercept would increase.

Question 30 (page 172)

C Correct. The number of potatoes harvested varies directly with the number of potato plants grown. Use the equation $y = kx$ to represent a direct proportion.

In this equation, y is the number of potatoes harvested, and x is the number of potato plants grown.

Last year's harvest was 189 potatoes from 9 potato plants. Substitute and solve for k, the proportionality constant.

$$y = kx$$
$$189 = k(9)$$
$$\frac{189}{9} = \frac{k(9)}{9}$$
$$k = 21$$

The equation describing this situation is $y = 21x$.

Find the number of potatoes to be harvested from 14 plants.

Substitute 14 for x in the equation $y = 21x$. Solve the equation for y.

$$y = 21x$$
$$y = 21(14)$$
$$y = 294$$

The gardener can expect to harvest 294 potatoes this year.

Objective 4

Question 31 (page 181)

B Correct. Jemma's dinner cost \$5 more than her cousin's. Represent the cost of her cousin's dinner with x. Represent the cost of Jemma's dinner with $x + 5$. Their combined bill was under \$25.

Write an inequality showing that the cost of the cousin's dinner plus the cost of Jemma's dinner was under \$25.

$$x + (x + 5) < 25$$

Question 32 (page 181)

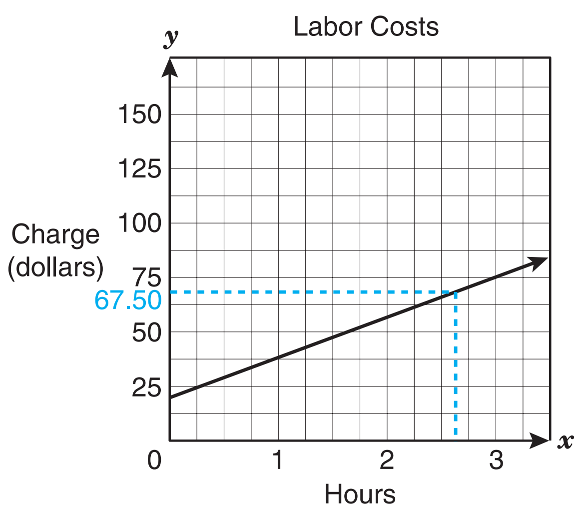
A Correct. The current population of Grandville is 15,400 people. Each year the population increases by 325 people. Let n represent the number of years it will take for the population to reach 18,000 people. In n years it will increase by $325n$ people.

Write an equation that shows this relationship.

$$15{,}400 + 325n = 18{,}000$$

Question 33 (page 181)

D Correct. Use the graph to match the labor charge with the number of hours the mechanic worked.



The labor charge was \$67.50. Find that value on the y-axis. Go across to the graph at this y-value and find the corresponding x-axis value for that point. The x-value is between $h = 2.50$ and $h = 2.75$. Therefore, $2.50 < h < 2.75$.

Question 34 (page 181)

C Correct. Represent the dimensions of the original rectangle. Its width is w. Its length is 6 feet greater than its width. Its length can be represented by $w + 6$.

The new rectangle has the same length, $w + 6$. Its width is 1 foot larger, $w + 1$.

For the new rectangle the length is twice the width. Represent this relationship with an equation.

$$\begin{aligned} \text{length} &= \text{twice the width} \\ w + 6 &= 2(w + 1) \\ w + 6 &= 2w + 2 \\ -w \quad &= -w \\ \hline 6 &= w + 2 \\ 4 &= w \end{aligned}$$

The width of the rectangle is 4 feet. Its length is 6 feet greater, or $4 + 6 = 10$ feet.

Question 35 (page 182)

B Correct.

The distance, d, Karen's car can go before it runs out of gas is equal to the capacity of the tank, 15 gallons, times the number of miles the car gets per gallon, n.

$$d = 15n$$

The number of miles her car gets per gallon is between 25 miles and 30 miles. Since the question asks for the least possible number of miles Karen can drive before she runs out of gas, choose the least possible value for n, 25.

Substitute this value into the equation.

$$d = (15)(25)$$
$$d = 375$$

Karen can drive at least 375 miles before she runs out of gas.

Question 36 (page 182)

B Correct. Represent the number of sheets Mrs. Green bought with s. She bought twice as many pillowcases as sheets. Represent the number of pillowcases she bought with $2s$.

The sheets cost \$5 each. The cost of the sheets Mrs. Green bought is equal to the number of sheets times the price per sheet. Represent the cost of the sheets she bought with $5s$.

The pillowcases cost \$2 each. The cost of the pillowcases Mrs. Green bought is equal to the number of pillowcases times the price per pillowcase. Represent the cost of the pillowcases she bought with $2(2s)$.

Write an inequality showing that the total amount Mrs. Green spent was less than \$40.

$$5s + 2(2s) < 40$$
$$5s + 4s < 40$$
$$9s < 40$$
$$s < 4.44$$

Mrs. Green bought, at most, 4 sheets. She bought twice as many pillowcases, or $2 \cdot 4 = 8$ pillowcases at most.

Question 37 (page 182)

A Correct. Represent the number of new stamps Hector has with h. Represent the number of new stamps Martha has with m. Together they have 32 new stamps. Represent this relationship with an equation.

$$\text{Hector} + \text{Martha} = 32$$
$$h + m = 32$$

Represent the number of new stamps they have after trading four stamps. Hector gives four stamps to Martha. Hector has four fewer stamps, $(h - 4)$. Martha has four more stamps, $(m + 4)$.

Martha now has three times as many stamps as Hector. Represent this relationship with an equation.

$$3 \cdot \text{Hector} = \text{Martha}$$
$$3(h - 4) = m + 4$$

The system of equations below would allow you to calculate the number of stamps Hector has and the number of stamps Martha has.

$$h + m = 32$$
$$3(h - 4) = m + 4$$

Question 38 (page 182)

D Correct. Represent the number of miles John ran home with r. Represent the number of miles he walked home with w. If John lives 3.5 miles from school, then the number of miles he walked plus the number of miles he ran must equal 3.5 miles.

$$r + w = 3.5$$

John walked six times as far as he ran.

$$w = 6r$$

The system of equations below can be used to find how many miles he ran.

$$w + r = 3.5$$
$$w = 6r$$

Objective 5

Question 39 (page 190)

B Correct. The function $g(x)$ is quadratic. Its parent function is $y = x^2$. It is in the form $g(x) = x^2 + c$, where c indicates how many units the parent function has been translated up or down. The vertex of $g(x)$ is 8 units below the origin, so the value of c is -8. The function can be written as $g(x) = x^2 - 8$.

Question 40 (page 190)

A Correct. The vertex of the graph of $f(x) = x^2 - 7$ is 7 units below the origin, and the vertex of $g(x) = x^2 + 5$ is 5 units above the origin. The distance between the vertices is 12 units. The vertex of $f(x)$ is 12 units below the vertex of $g(x)$.

Question 41 (page 190)

C Correct. Substitute the given expression into the formula for the area of a square.

$$A = s^2$$
$$A = (4x^3yz^4)^2$$

When raising a term with an exponent to a power, multiply the exponents.

$$A = (4)^2(x^3)^2y^2(z^4)^2$$
$$A = 4^2x^{3 \cdot 2}y^2z^{4 \cdot 2}$$
$$A = 16x^6y^2z^8$$

The area of the square is $16x^6y^2z^8$ square units.

Question 42 (page 190)

A Correct. The formula for the area of a parallelogram is $A = bh$. Solve for h.

$$A = bh$$
$$\frac{A}{b} = \frac{bh}{b}$$
$$\frac{A}{b} = h$$

Substitute the given expressions into the formula.

$$\frac{A}{b} = h$$
$$\frac{35p^6q^6}{5pq^2} = h$$
$$7p^{6-1}q^{6-2} = h$$
$$7p^5q^4 = h$$

The height of the parallelogram is $7p^5q^4$ units.

MATHEMATICS

Objective 6

Question 43 (page 204)

A Incorrect. If one quadrilateral is a dilation of the other, then the two quadrilaterals are similar. The lengths of the corresponding sides of similar figures must be proportional.

Compare the ratios of the lengths of the corresponding sides.

$$\frac{3}{3} \neq \frac{3}{5} \neq \frac{5}{5} \neq \frac{7}{8}$$

They are not proportional.

B Incorrect. If one quadrilateral is a dilation of the other, then the two quadrilaterals are similar. The lengths of the corresponding sides of similar figures must be proportional.

Compare the ratios of the lengths of the corresponding sides.

$$\frac{3}{4} = \frac{3}{4} \neq \frac{5}{6} \neq \frac{7}{9}$$

They are not proportional.

C Correct. If one quadrilateral is a dilation of the other, then the two quadrilaterals are similar. The lengths of the corresponding sides of similar figures must be proportional. Only the side lengths in choice C form a proportional relationship with the given side lengths.

$$\frac{3}{6} = \frac{3}{6} = \frac{5}{10} = \frac{7}{14}$$

They are proportional; all the ratios equal $\frac{1}{2}$.

D Incorrect. If one quadrilateral is a dilation of the other, then the two quadrilaterals are similar. The lengths of the corresponding sides of similar figures must be proportional.

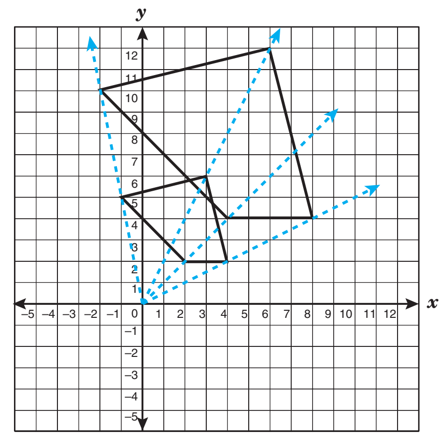
Compare the ratios of the lengths of the corresponding sides.

$$\frac{3}{9} = \frac{3}{9} = \frac{5}{15} \neq \frac{7}{24}$$

They are not proportional.

Question 44 (page 204)

B Correct. A figure has been dilated if it has the same shape, but its size has changed. Since the two quadrilaterals have the same shape but different sizes, this is a dilation.



Question 45 (page 205)

A Correct. The coordinates of the vertices of $LMNP$ are L (0, 2), M (4, 2), N (4, 0), and P (0, 0).

The length of $\overline{NP}$ is the difference between the x-coordinates of N and P.

$$4 - 0 = 4$$

The length of $\overline{LP}$ is the difference between the y-coordinates of L and P.

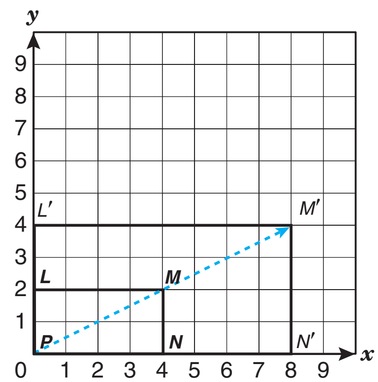
$$2 - 0 = 2$$

The scale factor of the dilation is 2. The length of $\overline{N'P}$ should be 8 units, and the length of $\overline{L'P}$ should be 4 units.

The coordinates of the vertices of $L'M'N'P$ in choice A are L' (0, 4), M' (8, 4), N' (8, 0), and P (0, 0).

$$N'P = 8 - 0 = 8$$
$$L'P = 4 - 0 = 4$$

Only the coordinates in choice A produce side lengths showing the correct scale factor, 2.



MATHEMATICS

Question 46 (page 206)

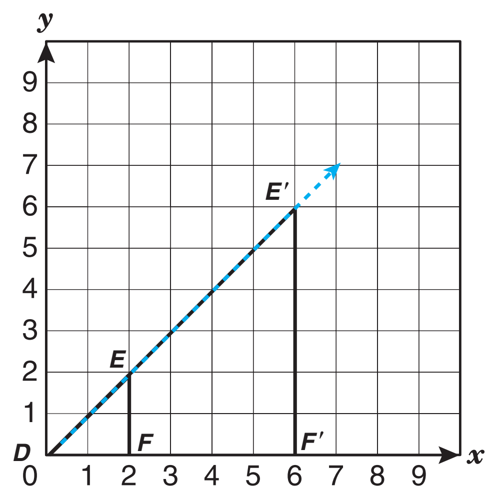
D Correct. The coordinates of point N are (5, 4).

Because this is a reflection across the x-axis, the x-coordinate of point N does not change. The x-coordinate of point N is 5, so the x-coordinate of point N' is also 5.

The y-coordinate of point N is 4. Point N is 4 units above the x-axis, so N' will be 4 units below the x-axis. The y-coordinate of point N' is -4. The coordinates of N' are $(5, -4)$.

Question 47 (page 206)

D Correct. $\overline{DF}$ is a horizontal segment with a length of 2 units. $\overline{DF'}$ will also be a horizontal segment, but it will have a length of $2 \cdot 3 = 6$ units. So the coordinates of F' will be (6, 0). $\overline{EF}$ is a vertical segment with a length of 2 units. $\overline{E'F'}$ will also be a vertical segment, but it will have a length of $2 \cdot 3 = 6$ units. So the coordinates of E' will be (6, 6).



Question 48 (page 207)

A Correct. The vertices of the original triangle are $(-6, 2)$, $(-1, 3)$, and $(-1, -1)$.

If the triangle is translated 4 units to the right, then 4 must be added to the x-coordinate of each vertex.

$$-6 + 4 = -2$$
$$-1 + 4 = 3$$
$$-1 + 4 = 3$$

If the triangle is translated 2 units down, then 2 must be subtracted from the y-coordinate of each vertex.

$$2 - 2 = 0$$
$$3 - 2 = 1$$
$$-1 - 2 = -3$$

The vertices of the translated figure are $(-2, 0)$, (3, 1), and $(3, -3)$.

Only the vertices of the triangle in choice A have these coordinates.

Question 49 (page 208)

D Correct. The midpoint of a line segment is found by averaging the x- and y-coordinates. Represent the coordinates of the endpoint R with x and y.

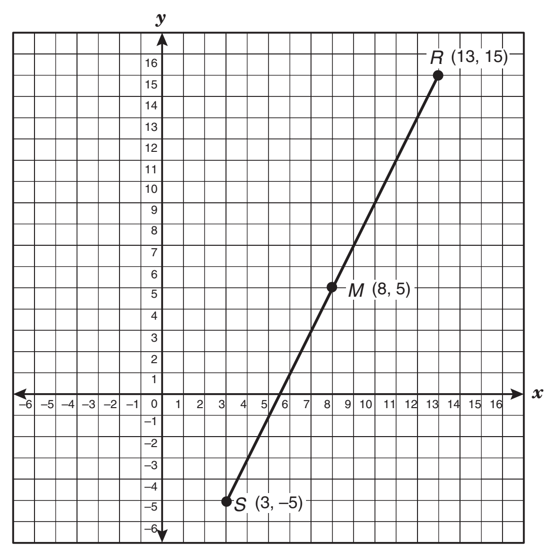
S $(3, -5)$

R (x, y)

The midpoint of $\overline{RS}$ is (8, 5).

$$\frac{3 + x}{2} = 8 \qquad \frac{-5 + y}{2} = 5$$
$$3 + x = 16 \qquad -5 + y = 10$$
$$x = 13 \qquad y = 15$$

So point R has the coordinates (13, 15).



Question 50 (page 208)

C Correct. Point P is 3.5 units to the right of the origin and 2.5 units down. The coordinates of point P are $(3.5, -2.5)$.

Question 51 (page 208)

A Incorrect. The coordinates of point R are $(-5, 0)$. Check to see whether these coordinates satisfy the given conditions.

The x-coordinate of point R is -5. Is $-5 < -1$? Yes.

The y-coordinate of point R is 0. Is $0 > 2$? No.

The coordinates of point R do not satisfy both conditions.

B Correct. The coordinates of point S are $(-3, 5)$. Check to see whether these coordinates satisfy the given conditions.

The x-coordinate of point S is -3. Is $-3 < -1$? Yes.

The y-coordinate of point S is 5. Is $5 > 2$? Yes.

The coordinates of point S satisfy both conditions.

C Incorrect. The coordinates of point T are $(0, 6)$. Check to see whether these coordinates satisfy the given conditions.

The x-coordinate of point T is 0. Is $0 < -1$? No.

The y-coordinate of point T is 6. Is $6 > 2$? Yes.

The coordinates of point T do not satisfy both conditions.

D Incorrect. The coordinates of point V are $(3, -2)$. Check to see whether these coordinates satisfy the given conditions.

The x-coordinate of point V is 3. Is $3 < -1$? No.

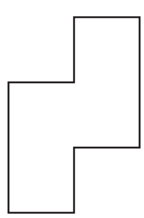
The y-coordinate of point V is -2. Is $-2 > 2$? No.

The coordinates of point V do not satisfy either condition.

Objective 7

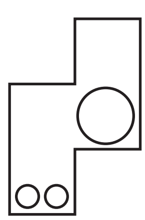
Question 52 (page 216)

B Correct. The model can be viewed as being made up of three rectangular prisms. The front and the back prisms are at 90° angles to the center prism. Seen from above, they form a figure like the one below.



On top of the front prism are two small cylinders. Seen from above, the cylinders will appear as circles.

Atop the center prism is a large cylinder that almost completely covers it.



The back prism has nothing on top of it.

Only choice B shows all the elements placed correctly.

Question 53 (page 217)

C Correct. When the figure in choice C is viewed from above, it appears as a row of four squares. When it is viewed from the side, it appears as a column of two squares. When it is viewed from the front, it appears as a row of four squares with an extra square stacked on either end. This figure matches the top, front, and side views given in the question.

Question 54 (page 218)

A Correct.
Find the total area of the windows. Use the formula for the area of a rectangle to find the area of one window.

$$A = lw$$
$$A = 6 \cdot 5 = 30 \text{ ft}^2$$

The area of one window is 30 square feet. There are 60 windows in all, so multiply by 60 to find the total area of the windows.

$$60 \cdot 30 = 1{,}800 \text{ ft}^2$$

To find the fraction of the building's walls taken up by windows, divide the area of the windows (1,800 square feet) by the total area of the walls (18,000 square feet).

$$\frac{1{,}800}{18{,}000} = \frac{1}{10}$$

Question 55 (page 218)

The correct answer is 67.5. There are 360° in a circle. Since the octagon is regular, all 8 of the angles at its center are equal in measure. First find the measure of $\angle B$.

$$m\angle B = 360° \div 8$$
$$m\angle B = 45°$$

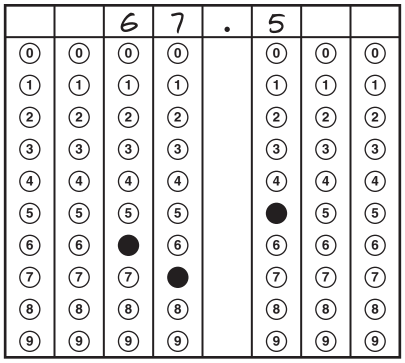
The sum of the measures of the angles of a triangle is 180°.

$$m\angle A + m\angle B + m\angle C = 180°$$
$$m\angle A + 45° + m\angle C = 180°$$
$$m\angle A + m\angle C = 135°$$

Since the triangle is isosceles, $m\angle A$ and $m\angle C$ are equal.

$$\frac{135}{2} = 67.5$$

The measure of $\angle A$ is 67.5°.



Question 56 (page 219)

C Correct.

The side lengths of a right triangle satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$. The longest side of the triangle is 2.5 units long, so it should be the hypotenuse. The legs are 1.5 units and 2 units long.

The square formed by the hypotenuse is 2.5 units by 2.5 units. It has an area of $2.5 \cdot 2.5 = 6.25$ square units.

The square formed by the shorter leg is 1.5 units by 1.5 units. It has an area of $1.5 \cdot 1.5 = 2.25$ square units.

The square formed by the longer leg is 2 units by 2 units. It has an area of $2 \cdot 2 = 4$ square units.

Since $6.25 = 2.25 + 4$, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs. The triangle is a right triangle. The values for a, b, and c in choice C satisfy the Pythagorean Theorem; they form a right triangle.

Question 57 (page 220)

B Correct.

You are given the length of the shelf paper (9 feet). Find the number of inches in 9 feet. Use the measurement fact 12 inches = 1 foot.

$$9 \text{ ft} \cdot 12 \text{ in. per ft} = 108 \text{ in.}$$

Tran has 108 inches of shelf paper to start with and 40.5 inches left when he finishes the job. Find the number of inches of paper he uses to cover the shelves.

$$108 - 40.5 = 67.5 \text{ in.}$$

To find the length of each shelf, divide the total length of the shelves (67.5 inches) by the number of shelves.

$$67.5 \div 3 = 22.5 \text{ in.}$$

Each shelf is 22.5 inches long.

Objective 8

Question 58 (page 240)

B Correct.

Use the formula in the Mathematics Chart for the total surface area of a cylinder.

$$S = 2\pi rh + 2\pi r^2$$

The radius, r, of the cylinder is 4 inches; its height, h, is 12 inches. Substitute these values into the formula and solve for the surface area, S.

$$S = 2\pi \cdot 4 \cdot 12 + 2\pi \cdot 4^2$$
$$S = 96\pi + 32\pi$$
$$S = 128\pi \text{ in.}^2$$

The total surface area of the cylinder is 128π square inches.

Question 59 (page 240)

C Correct. The large can is an enlargement of the regular can by a scale factor of 2. The volume of the regular can (72 cubic inches) multiplied by the cube of the scale factor (2^3) equals the volume of the large can.

$$V = 72 \cdot 2^3$$
$$V = 72 \cdot 8$$
$$V = 576 \text{ in.}^3$$

The volume of the large can is 576 cubic inches.

Question 60 (page 240)

The correct answer is 162.

Use the formula for the volume of a pyramid.

$$V = \frac{1}{3}Bh$$

In this formula B represents the area of the pyramid's base, and h represents the pyramid's height.

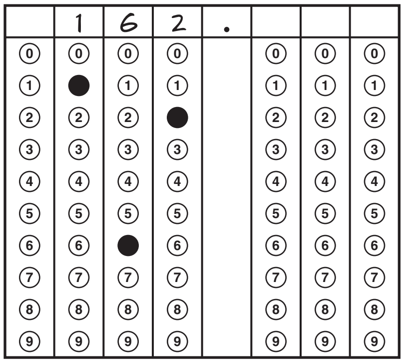
Find the area of the base, B. The base of a square pyramid is a square. Use the formula for the area of a square to find the area of the base.

$$A = s^2$$
$$A = 9^2$$
$$A = 81 \text{ ft}^2$$

The area of the pyramid's base, B, is 81 square feet. The pyramid's height, h, is 6 feet. Substitute the values for B and h into the volume formula.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3} \cdot 81 \cdot 6$$
$$V = 162 \text{ ft.}^3$$

The volume of the pyramid is 162 cubic feet.



Question 61 (page 240)

C Correct.

The length of the smaller frame corresponds to the length of the larger frame, and the width of the smaller frame corresponds to the width of the larger frame. The ratios of these corresponding sides are equal.

$$\frac{\text{length}_{\text{smaller}}}{\text{length}_{\text{larger}}} = \frac{\text{width}_{\text{smaller}}}{\text{width}_{\text{larger}}}$$

Let l represent the length of the larger frame. Substitute the given measurements in the proportion.

$$\frac{14}{l} = \frac{11}{24}$$

Use cross products to solve for the length of the larger frame.

$$11l = 14 \cdot 24$$
$$11l = 336$$
$$l \approx 30.5 \text{ in.}$$

The length of the larger frame is about 30.5 inches.

Question 62 (page 241)

D Correct.

The lengths of the two legs of the triangle are given. Use the Pythagorean Theorem to find the hypotenuse of the triangle, the distance between the sailboat and the buoy.

Let 1.8 equal a. Let 2.4 equal b.

$$a^2 + b^2 = c^2$$
$$(1.8)^2 + (2.4)^2 = c^2$$
$$3.24 + 5.76 = c^2$$
$$9 = c^2$$
$$3 = c$$

The distance between the sailboat and the buoy is 3 miles.

Question 63 (page 241)

B Correct. In similar triangles the lengths of corresponding sides are proportional.

$\overline{RS}$ corresponds to $\overline{VU}$ and $\overline{RT}$ corresponds to $\overline{VT}$.

Write a proportion.

$$\frac{RS}{VU} = \frac{RT}{VT}$$

Substitute the given measurements into the proportion.

$$\frac{RS}{24} = \frac{7.2}{18}$$

Use cross products to solve for the length of $\overline{RS}$.

$$18 \cdot RS = 24 \cdot 7.2$$
$$18 \cdot RS = 172.8$$
$$RS = 9.6 \text{ mm}$$

The length of $\overline{RS}$ is 9.6 millimeters.

Question 64 (page 241)

A Correct.

The volume of water that the pipe can hold is equal to the volume of the inner cylinder. Use the formula in the Mathematics Chart for the volume of a cylinder.

$$V = Bh$$

In this formula B represents the area of the cylinder's base, and h represents the cylinder's height.

Find the area of the base, B. The base of a cylinder is a circle. Use the formula for the area of a circle, $A = \pi r^2$. The radius, r, of the inner circle is equal to half its diameter.

$$3 \div 2 = 1.5$$
$$A = \pi r^2$$
$$A = \pi(1.5)^2$$
$$A \approx 7.065 \text{ cm}^2$$

The area of the base, B, is about 7.065 square centimeters. The cylinder's height, h, is 50 centimeters. Substitute the values for B and h into the volume formula.

$$V = Bh$$
$$V \approx 7.065 \cdot 50$$
$$V \approx 353.25 \text{ cm}^3$$

The volume of the cylindrical pipe is about 353 cubic centimeters, so it can hold about 353 cubic centimeters of water.

Question 65 (page 242)

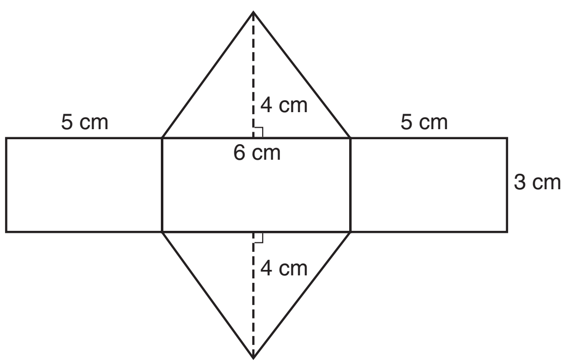
A Correct. If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their areas will be in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$. The ratio of the area of the smaller garden to the area of the larger garden is $\frac{1}{4}$. The areas are in the ratio $\frac{1}{4} = \frac{1^2}{2^2} = \left(\frac{1}{2}\right)^2$. Therefore, the dimensions of the gardens are in the ratio $\frac{1}{2}$.

Question 66 (page 242)

A Correct. The perimeter of the original square will increase by the scale factor of the dilation. The scale factor of the dilation is 3 because the square's dimensions are tripled. Therefore, the perimeter will increase by a factor of 3. The perimeter of the new square will be 3 times the perimeter of the original square.

Question 67 (page 242) 

D Correct.
To find the surface area of the prism, first measure the dimensions of the faces.



Then find the area of each of the five faces. The formula for the area of a rectangle is $A = lw$; the formula for the area of a triangle is $A = \frac{1}{2}bh$.

The center rectangle measures 6 centimeters by 3 centimeters.

$$A = 6 \cdot 3 = 18 \text{ cm}^2$$

This face has an area of 18 square centimeters.

The rectangles on the left and right each measure 5 centimeters by 3 centimeters.

$$A = 5 \cdot 3 = 15 \text{ cm}^2$$

Each of these faces has an area of 15 square centimeters.

The two triangles have bases of 6 centimeters and heights of 4 centimeters.

$$A = \frac{1}{2}(6 \cdot 4) = 12 \text{ cm}^2$$

Each of these faces has an area of 12 square centimeters.

The total surface area of the prism is the sum of the areas of its 5 faces.

$$S = 18 + 15 + 15 + 12 + 12 = 72 \text{ cm}^2$$

The total surface area of the triangular prism is 72 square centimeters.

Objective 9

Question 68 (page 264) 

D Correct.
The picture frame will be $10\frac{1}{4}$ inches long by $8\frac{5}{16}$ inches wide.

Use the formula for the perimeter of a rectangle.

$$P = 2l + 2w$$

$$P = 2 \cdot 10\frac{1}{4} + 2 \cdot 8\frac{5}{16}$$

$$P = 2 \cdot \frac{41}{4} + 2 \cdot \frac{133}{16}$$

$$P = \frac{41}{2} + \frac{133}{8}$$

$$P = \frac{41}{2} \cdot \frac{4}{4} + \frac{133}{8}$$

$$P = \frac{164}{8} + \frac{133}{8}$$

$$P = \frac{297}{8}$$

$$P = 37\frac{1}{8} \text{ in.}$$

The picture frame will use approximately $37\frac{1}{8}$ inches of molding. To find the number of inches remaining, subtract $37\frac{1}{8}$ inches from 4 feet. First convert 4 feet to inches by multiplying by 12.

$$4 \cdot 12 = 48$$

$$48 - 37\frac{1}{8} = \left(47\frac{8}{8}\right) - \left(37\frac{1}{8}\right) = 10\frac{7}{8}$$

There will be $10\frac{7}{8}$ inches of molding remaining from the original 4-foot length of molding.

Question 69 (page 264)

D Correct. Use a proportion to find 25% of $85.

$$\frac{25}{100} = \frac{n}{85}$$

Use cross products to solve for n.

$$100n = 25 \cdot 85$$
$$100n = 2125$$
$$n = 21.25$$

The discount on the skateboard is $21.25.

Subtract the amount of the discount from the original price to find the sale price.

$$85.00 - 21.25 = 63.75$$

The sale price of the skateboard is $63.75.

Question 70 (page 264)

B Correct. Write a ratio that shows the number of miles Janine can travel on 6 gallons.

$$\frac{192 \text{ mi}}{6 \text{ gal}}$$

Let n equal the distance she can travel on 10 gallons. Write a ratio that shows the number of miles she can travel on 10 gallons.

$$\frac{n}{10 \text{ gal}}$$

Write a proportion.

$$\frac{192 \text{ mi}}{6 \text{ gal}} = \frac{n}{10 \text{ gal}}$$
$$6n = 192 \cdot 10$$
$$6n = 1920$$
$$n = 320$$

Janine can travel 320 miles on 10 gallons of gasoline.

Question 71 (page 264)

C Correct. There are 20 bagels in the box, or 20 possible outcomes.

Since 4 bagels contain blueberries and 5 bagels contain cranberries, there are 9 possible outcomes that result in a bagel that contains either blueberries or cranberries. There are $20 - 9 = 11$ possible outcomes that result in a bagel that contains neither blueberries nor cranberries.

Therefore, the probability of choosing a bagel that contains neither blueberries nor cranberries is $\frac{11}{20}$. The fraction $\frac{11}{20}$ can be written as a percent.

$$\frac{11}{20} = \frac{x}{100}$$
$$20x = 11 \cdot 100$$
$$20x = 1100$$
$$x = 55$$

The probability of choosing a bagel that contains neither blueberries nor cranberries is 55%.

Question 72 (page 265)

C Correct. The outcome of the first trial of this experiment affects the likelihood of a favorable outcome in the second trial. The two events are dependent.

Find the probability of choosing a red apple on the first draw. There are 4 favorable outcomes. There are 10 possible outcomes. The probability of choosing a red apple on the first draw is $P(\text{red}_{\text{first}}) = \frac{4}{10} = \frac{2}{5}$.

Find the probability of choosing a green apple on the second draw. There are 6 favorable outcomes. There are now only 9 apples in the bag, so there are only 9 possible outcomes for the second draw. The probability of choosing a green apple on the second draw is $P(\text{green}_{\text{second}}) = \frac{6}{9} = \frac{2}{3}$.

Find the probability of choosing a red apple and then a green apple.

$$P(\text{red}_{\text{first}} \text{ and green}_{\text{second}}) = P(\text{red}_{\text{first}}) \cdot P(\text{green}_{\text{second}})$$
$$= \frac{2}{5} \cdot \frac{2}{3}$$
$$= \frac{4}{15}$$

The probability of choosing a red apple and then a green apple is $\frac{4}{15}$.

Question 73 (page 265)

A Incorrect. To determine the mean, find the sum of the scores and divide by the number of scores.

$$\frac{3 + 3 + 6 + 7 + 8}{5} = \frac{27}{5} = 5.4$$

The mean of the scores is 5.4. It is not the highest score.

B Correct. To determine the median, list the scores from least to greatest: 3, 3, 6, 7, and 8. The median is the middle number in this list, 6. Using the median would give Luther the highest final score.

C Incorrect. The mode is the score that appears most often in the list of scores, 3. It is not the highest score.

D Incorrect. To determine the range, subtract the lowest score from the highest score.

$$8 - 3 = 5$$

The range of the scores is 5. It is not the highest score.

Question 74 (page 265)

B Correct. Find the experimental probability that a customer will rent an action movie. The number of customers preferring action movies in the sample is 18. The total number of customers in the sample is $12 + 20 + 18 = 50$. The experimental probability that a customer will rent an action movie is $\frac{18}{50} = \frac{9}{25}$.

Use a proportion to find the number of action-movie rentals.

Let n equal the number of action movies rented out of 425 movies.

Write a proportion.

$$\frac{n}{425} = \frac{9}{25}$$
$$25n = 425 \cdot 9$$
$$25n = 3825$$
$$n = 153$$

The store can expect 153 action movies to be rented on a day when 425 movies are checked out.

Question 75 (page 266)

A Correct. The histograms show that scores between 70 and 79 receive a C, scores between 80 and 89 receive a B, and scores between 90 and 99 receive an A. There are 4 scores that receive a C: 75, 71, 77, and 78. There are 8 scores that receive a B: 81, 88, 85, 85, 89, 80, 83, and 81. There are 6 scores that receive an A: 94, 92, 98, 91, 91, and 90.

Only choice A shows 4 C's, 8 B's, and 6 A's.

Question 76 (page 267)

C Correct. Add the values in the table to find Paul's total average monthly expenses.

$$570 + 380 + 190 + 285 + 475 = 1{,}900$$

Find the percentage of Paul's expenses that represents his rent.

$$\frac{570}{1{,}900} = \frac{n}{100}$$
$$1{,}900n = 570 \cdot 100$$
$$1{,}900n = 57{,}000$$
$$n = 30$$

About 30% of Paul's expenses represents his rent. Only choices B and C fit this requirement.

Choices B and C both show the same percentage for Paul's car payment.

Find the percentage of Paul's expenses that represents utilities.

$$\frac{190}{1{,}900} = \frac{n}{100}$$
$$1{,}900n = 190 \cdot 100$$
$$1{,}900n = 19{,}000$$
$$n = 10$$

About 10% of Paul's expenses represents utilities. Only choice C fits this requirement.

Question 77 (page 268)

A Incorrect. Sales of Fabulous CDs increased from January to March and then decreased from March to June.

B Incorrect. In June, sales of Ransom CDs were greater than sales of Fabulous CDs.

C Incorrect. The least sales for Fabulous occurred in January (about 1 million), and the greatest sales occurred in March (about 9.5 million). Subtract these values to estimate the range.

$$9.5 \text{ million} - 1 \text{ million} = 8.5 \text{ million}$$

The range of Fabulous CD sales is about 8.5 million CDs.

The least sales for Ransom occurred in January (about 0.5 million), and the greatest sales occurred in June (about 9.7 million). Estimate the range.

$$9.7 \text{ million} - 0.5 \text{ million} = 9.2 \text{ million}$$

The range of Ransom CD sales is about 9.2 million CDs, which is greater than the range of Fabulous CD sales.

D Correct. Arranged from least to greatest, the monthly CD sales for Fabulous are January, February, June, May, April, and March. The two middle months in this list are June (about 2.1 million) and May (about 5.2 million). To find the median, find the mean of these two sales numbers.

$$\frac{2.1 + 5.2}{2} = \frac{7.3}{2} = 3.65$$

The median of the monthly CD sales for Fabulous is about 3.65 million.

Arranged from least to greatest, the monthly CD sales for Ransom are January, February, March, April, May, and June. The two middle months in this list are March (about 2.3 million) and April

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(about 2.8 million). To find the median, find the mean of these two sales numbers.

$$\frac{2.3 + 2.8}{2} = \frac{5.1}{2} = 2.55$$

The median of the monthly CD sales for Ransom is about 2.55 million.

The median of the monthly CD sales for Ransom is less than the median monthly CD sales for Fabulous.

Objective 10

Question 78 (page 284)

C Correct. The clerk's earnings are equal to the number of hours he worked multiplied by his hourly pay rate of \$7.00, plus the number of large-screen televisions he sold multiplied by \$15.00. The number of hours the clerk worked can be found by adding up the values in the table. The number of large-screen televisions sold by the clerk is not given. This information must be provided in order to solve the problem.

Question 79 (page 284)

D Correct.

Julia's actual writing rate is 38 minutes per page. Based on this rate, estimate the time it will take her to write the report.

$$\frac{38 \text{ min}}{1 \text{ page}} = \frac{n \text{ min}}{4 \text{ pages}}$$

$$1n = 38 \cdot 4$$

$$n = 152$$

If she keeps writing the report at this rate, it will take her 152 minutes to complete the report. Subtract this time from 4 hours, her original estimate, to determine by how many hours and minutes Julia overestimated the total amount of time required to write the report.

$$4 \text{ h} - 152 \text{ min} =$$

$$4 \cdot 60 \text{ min} - 152 \text{ min} =$$

$$240 \text{ min} - 152 \text{ min} = 88 \text{ min}$$

Divide by 60 to find the number of hours and minutes.

$$88 \div 60 = 1 \text{ R}28$$

$$88 \text{ minutes} = 1 \text{ hour } 28 \text{ minutes}$$

Julia overestimated the amount of time it would take her to write the report by 1 hour 28 minutes.

Question 80 (page 284)

C Correct.

The function $y = 4x + 1$ is a linear function. Therefore, its equation is in the form $y = mx + b$, where m represents the slope and b represents the y-intercept. The slope of the original graph is 4.

The slope of the graph increases by 25%. Use a proportion to find 25% of 4.

$$\frac{25}{100} = \frac{x}{4}$$

$$100x = 25 \cdot 4$$

$$100x = 100$$

$$x = 1$$

The slope of the new graph is 1 more than the slope of the original graph: $4 + 1 = 5$. The slope of the new line is 5. Its equation is in the form $y = mx + b$. For the new line, $m = 5$.

The point (2, 8) is on the graph. Substitute these values to find b.

$$y = 5x + b$$

$$8 = 5 \cdot 2 + b$$

$$8 = 10 + b$$

$$-2 = b$$

The y-intercept, b, of the new graph is -2. Substitute $m = 5$ and $b = -2$ in the slope-intercept form of the equation, $y = mx + b$. The equation of the new line is $y = 5x - 2$.

Question 81 (page 284)

B Correct.

The area of the original rectangle is equal to its length times its width.

$$A = lw$$

$$A = 3 \cdot 4$$

$$A = 12$$

The area of the original rectangle is 12 square units.

If a figure is enlarged by a scale factor, then its area will increase by the square of that scale factor. The rectangle is enlarged by a scale factor of 1.3. Its area should increase by a factor of $(1.3)^2 = 1.69$.

Multiply by 1.69 to find the area of the enlarged rectangle. The area of the enlarged rectangle is $12 \cdot 1.69 = 20.28$ square units. The area of the rectangle increases by $20.28 - 12 = 8.28$ square units.

The percent increase is found by dividing the increase by the original area.

$$\frac{8.28}{12} = 0.69 = 69\%$$

The area increases by 69%.

Question 82 (page 285)

A Correct. One way to solve this problem is to make a table showing the number of games in each round of the tournament.

Round	Number of Teams at Start of Round	Number of Games	Number of Losing Teams	Number of Winning Teams
1	16	8	8 eliminated	8 go to round 2
2	8	4	4 eliminated	4 go to round 3
3	4	2	2 eliminated	2 go to round 4
4	2	1	1 eliminated	1 (champion)

Find the total number of games by adding the values in the third column of the table.

$$8 + 4 + 2 + 1 = 15$$

A total of 15 games are played during the tournament.

Question 83 (page 285)

B Correct.

The function $c = (9.15 + 4.25)n + 3{,}000$ can be used to find the cost of producing 500 rackets. Substitute 500 for n.

$$c = (9.15 + 4.25)n + 3{,}000$$
$$c = (9.15 + 4.25)500 + 3{,}000$$
$$c = (13.4)500 + 3{,}000$$
$$c = 6{,}700 + 3{,}000$$
$$c = 9{,}700$$

If 500 rackets are sold for \$25 each, they will generate 500 • \$25 = \$12,500 in revenue. The profit on the sale of 500 rackets is the difference between the cost of producing them and the sales revenue they generate.

$$\$12{,}500 - \$9{,}700 = \$2{,}800$$

Question 84 (page 285)

A Correct. Let x equal the cost per pair of socks. The cost of the socks is equal to the number of pairs (5) times the price per pair (x), or $5x$. The cost of the shoes plus the cost of the socks is equal to the total cost.

$$\$65 + 5x = \$100$$

This equation is equivalent to $5x + 65 = 100$.

B Incorrect. The interest on the loan (\$65) is equal to the principal (\$100) times the simple interest rate per year (5%) times the time in years. Let x equal the time in years.

$$\$65 = \$100 \cdot 0.05x$$

This equation is not equivalent to $5x + 65 = 100$.

C Incorrect. Let x equal the number of hours needed to ride 100 miles. To solve this problem, use a proportion.

$$\frac{65 \text{ miles}}{5 \text{ hours}} = \frac{100 \text{ miles}}{x \text{ hours}}$$
$$65x = 5 \cdot 100$$

This equation is not equivalent to $5x + 65 = 100$.

D Incorrect. The amount left in the account is equal to the original amount (\$100) minus the withdrawal (\$65) and plus the deposit (\$5). Let x equal the amount left in the account.

$$x = \$100 - \$65 + \$5$$

This equation is not equivalent to $5x + 65 = 100$.

Question 85 (page 286)

C Correct. Let x equal the number of hours a bicycle is rented and let y equal the total cost of renting a bike. The total cost of renting a bike is equal to the hourly rate, \$4 per hour, times the number of hours it is rented, x, plus \$5. An equation representing this situation is $y = 4x + 5$. The equation is in slope-intercept form, $y = mx + b$, in which $m = 4$ and $b = 5$.

Find the equation of the graph.

The y-intercept, b, is 5. Use the points (0, 5) and (5, 25) to find m, the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{25 - 5}{5 - 0}$$
$$m = \frac{20}{5}$$
$$m = 4$$

The equation of the graph is $y = 4x + 5$. This equation matches the one that describes the total cost of renting a bike.

Question 86 (page 286)

D Correct. Look for a pattern in the sequence of graphs. The slope increases by 1 each time, and the y-intercept increases by 2.

Graph Number	Slope	y-intercept
1	1	1
2	2	3
3	3	5
4	4	7

The next graph should have a slope of 5 ($m = 5$) and a y-intercept of 9 ($b = 9$). In slope-intercept form its equation should be $y = 5x + 9$.

Question 87 (page 287)

B Correct.

Look for a pattern in the functions. The first function, $f(x) = 2x^2 + 1$, is in the form $y = ax^2 + bx + c$. It is a quadratic function. The second function, $2x + 3y = 6$, is in the form $Ax + By = C$. It is a linear function in standard form. The third function is presented as a graph. Its graph is a parabola; it is a quadratic function. The fourth function is presented as a table. Look for a pattern in the table.

x	x^2	y
-2	4	6
0	0	2
1	1	3
3	9	11

The y-values are 2 more than the squares of the x-values. This is a quadratic function.

Only the second function, $2x + 3y = 6$, is not a quadratic function. It does not fit the pattern.

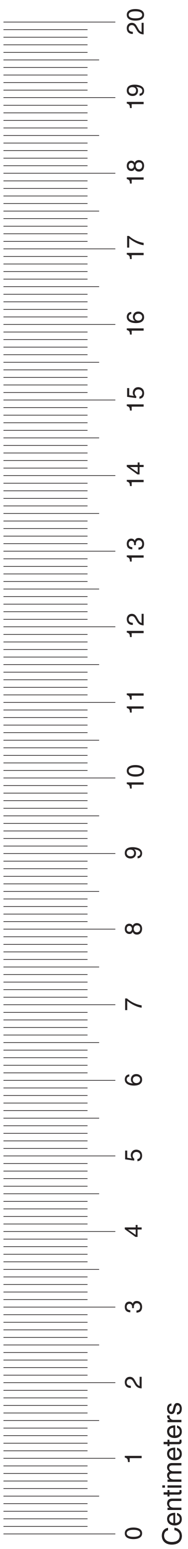
Question 88 (page 287)

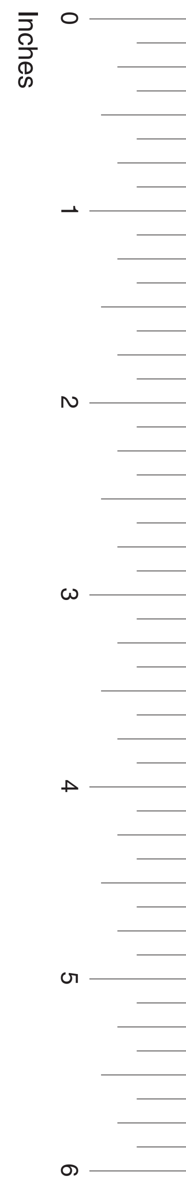
A Correct. The distributive property states that the factor outside the parentheses must multiply both terms inside the parentheses. In going from Step 1 to Step 2, the student should have multiplied both $2x$ and 6 by 3, not just $2x$.

Step 1: $3(2x + 6) - 4 = 14$

Step 2: $6x + 18 - 4 = 14$

Grades 9, 10, and 11 Exit Level
Mathematics Chart





LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

Continued on the next side

Grades 9, 10, and 11 Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$
Surface Area	cube	$S = 6s^2$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism or cylinder	$V = Bh$*
	pyramid or cone	$V = \frac{1}{3}Bh$*
	sphere	$V = \frac{4}{3}\pi r^3$
**B represents the area of the Base of a solid figure.*		
Pi	π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

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